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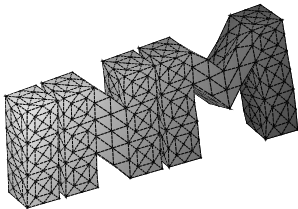
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7. Workshop on  
**Fast Boundary Element Methods in  
Industrial Applications**

Söllerhaus, 15.–18.10.2009

U. Langer, O. Steinbach, W. L. Wendland (eds.)

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**Berichte aus dem  
Institut für Numerische Mathematik**

Book of Abstracts 2009/7



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## Program

| Thursday, 15.10.2009 |                                                                                                                       |
|----------------------|-----------------------------------------------------------------------------------------------------------------------|
| 15.00–16.20          | Coffee                                                                                                                |
| 16.20–16.30          | Opening                                                                                                               |
| 16.30–17.00          | M. Bebendorf (Bonn)<br>Adaptive Cross Approximation of Trivariate Functions                                           |
| 17.00–17.30          | G. Riekh (Wien)<br>The use of frames in BEM                                                                           |
| 17.30–18.00          | L. Banjai (Leipzig)<br>Multistep and multistage convolution quadratures of the wave equation                          |
| 18.30                | Dinner                                                                                                                |
| Friday, 16.10.2009   |                                                                                                                       |
| 9.00–9.30            | A. Salvadori (Brescia)<br>A variational integral formulation for fracture mechanics                                   |
| 9.30–10.00           | W. Weber (Erlangen)<br>Non-linear stress analyzes of cracked structures by the boundary element method                |
| 10.00–10.30          | W. Kreuzer (Wien)<br>A BEM model of a tunnel in a layered orthotropic medium                                          |
| 10.30–11.00          | Coffee                                                                                                                |
| 11.00–11.30          | W. Lemster (Göttingen)<br>A MHD problem on unbounded domains: Coupling of FEM and BEM                                 |
| 11.30–12.00          | S. Engleder (Graz)<br>Boundary element methods for the eddy current model                                             |
| 12.00–12.30          | A. von Graefe (Hamburg)<br>The Rankine boundary element method for the calculation of the potential flow around ships |
| 12.30                | Lunch                                                                                                                 |
| 15.00–15.30          | Coffee                                                                                                                |
| 15.30–16.00          | S. Ferraz-Leite (Wien)<br>Convergence of adaptive BEM                                                                 |
| 16.00–16.30          | P. Goldenits (Wien)<br>Adaptive BEM for mixed boundary value problems                                                 |
| 16.30–17.00          | D. Praetorius (Wien)<br>ASBEST – Adaptive Symmetric Boundary Element Simulation Tool                                  |
| 17.00–17.30          | Break                                                                                                                 |
| 17.30–18.00          | G. Of (Graz)<br>BEM/FEM coupling for transient Maxwell problems                                                       |
| 18.00–18.30          | Ma. Messner (Graz)<br>A general purpose Fast BEM library for the solution of time domain elastodynamic problems       |
| 18.30                | Dinner                                                                                                                |

| Saturday, 17.10.2009 |                                                                                                                           |
|----------------------|---------------------------------------------------------------------------------------------------------------------------|
| 9.00–9.30            | C. Hofreither (Linz)<br>Boundary element based Trefftz methods for potential problems                                     |
| 9.30–10.00           | S. Weißer (Saarbrücken)<br>Adaptive FEM with local Trefftz trial functions for elliptic equations                         |
| 10.00–10.30          | M. Fleck (Saarbrücken)<br>BEM–based FEM for eddy current problems                                                         |
| 10.30–11.00          | Coffee                                                                                                                    |
| 11.00–11.30          | T. Klug (Baden)<br>Parallelization strategies in CASOPT                                                                   |
| 11.30–12.00          | M. Windisch (Graz)<br>Preconditioned BETI for Helmholtz                                                                   |
| 12.00–12.30          | O. Steinbach, P. Urthaler (Graz)<br>Boundary element methods in dielectric media                                          |
| 12.30                | Lunch                                                                                                                     |
| 13.30–17.00          | Hiking tour                                                                                                               |
| 17.30–18.00          | C. Pechstein (Linz)<br>Explicit constants for some boundary integral operators                                            |
| 18.00–18.30          | O. Steinbach (Graz)<br>Stable coupling of finite and boundary elements                                                    |
| 18.30                | Dinner                                                                                                                    |
| Sunday, 18.10.2009   |                                                                                                                           |
| 9.00–9.30            | Z. Andjelic (Baden)<br>Reactor simulation using BEM                                                                       |
| 9.30–10.00           | W. L. Wendland (Stuttgart)<br>Boundary integral equations for two–dimensional low Reynolds number flow past a porous body |
| 10.00–10.30          | L. Raguin (Zürich)<br>Spectral BIE methods for Engineering Moonlit Night at Nanoscale                                     |
| 10.30–11.00          | Coffee                                                                                                                    |

## Reactor simulation using BEM

Z. Andjelic

ABB Schweiz, Baden

Typical reactor components of the HV power systems are either fix shunt reactors or controllable shunt reactors. In this paper we shall illustrate the computational approach used for the simulation of the controllable shunt reactors used for compensation of the reactive power in energetic systems. There are several approaches how to achieve the controlling effects in the reactor. Here we analyze the orthogonal flux type controllable reactor using integral equation approach where by the controlling effect is achieved by the control of the saturation level of magnetic core. From the simulation point of view this type of problems has several peculiarities: complex structure of the controlling discs / windings, online changes of the saturation levels, online calculation of the inductances i.e. induced voltages etc.

In the paper we present the methodic for computation of the required field quantities using advanced integral approach. Although the treatment of the nonlinearities requires the volumetric meshing, the nice feature of this approach is that the matrix size is determined by the size of the surface discretization. The paper demonstrates usage of IEM for the computation of the inductances as a function of the DC current changes depending on the saturation levels of the magnetic material. The results are compared with the calculation results based on equivalent magnetic circuit calculation model.

# Multistep and multistage convolution quadratures of the wave equation

L. Banjai

Max-Planck Institute for Mathematics in the Sciences, Leipzig

We describe how a time-discretized wave equation in a homogeneous medium can be solved by boundary integral methods. The time discretization can be a multistep, Runge-Kutta, or a more general multistep-multistage method.

We describe an efficient, robust, and easily parallelizable method for solving the resulting discretized system that has the main advantages of time-stepping methods and of Fourier synthesis: at each time-step a system of linear equations with the same system matrix needs to be solved, yet computations can easily be done in parallel, the computational cost is almost linear in the number of time-steps, and only the Laplace transform of the time-domain fundamental solution is needed.

We will give results of a series of 3D experiments with a range of multistep and multistage time discretization methods: backward difference formula of order 2 (BDF2), Trapezoid rule, and the 3-stage Radau IIA methods are investigated in detail. The 3-stage Radau IIA method often performs overwhelmingly better than the multistep methods, especially for problems with many reflections, yet, in connection with hyperbolic problems backward difference formulas have so far been predominant in the literature on convolution quadrature.

We end with some comments and an outlook.



# **Adaptive Cross Approximation of Trivariate Functions**

M. Bebendorf  
Universität Bonn

We present a new scheme for the approximation of trivariate functions by sums of products of univariate functions. The method is based on the Adaptive Cross Approximation (ACA) initially designed for the approximation of bivariate functions.

## Boundary Element Methods for the Eddy Current Model

S. Engleder, O. Steinbach  
TU Graz

Magnetic Induction Tomography is a contactless imaging modality, which aims to obtain the conductivity distribution of the human body. The method is based on exciting the body by magnetic induction using an array of transmitting coils to induce eddy currents. A change of the conductivity distribution in the body results in a perturbed magnetic field, which can be measured as a voltage change in the receiving coils. Based on these measurements, the conductivity distribution can be reconstructed by solving an inverse problem.

The forward problem of this method can be described by the eddy current model. In this talk a boundary element method for this eddy current problem will be presented. The use of suitable preconditioners and fast boundary element methods will be discussed.

## Convergence of adaptive BEM

M. Aurada, S. Ferraz–Leite, D. Praetorius  
TU Wien

A posteriori error estimators and adaptive mesh-refinement have themselves proven to be an important tool for scientific computing. For error control in finite element methods (FEM), there is a broad variety of a posteriori error estimators available, and convergence as well as optimality of adaptive FEM is well-studied in the literature. This is in sharp contrast to the boundary element method (BEM). Although a posteriori error estimators and adaptive algorithms are also successfully applied to boundary element schemes, even convergence of adaptive BEM is hardly understood mathematically. In our contribution, we present and discuss recent mathematical results [1, 3] which give first positive answers for adaptive BEM.

As BEM model problem for our talk serves the weakly-singular integral equation

$$Vu = f$$

associated with the Laplace operator in 2D and 3D and stated in the energy space  $\tilde{H}^{-1/2}(\Gamma)$ . We use the lowest-order Galerkin method with piecewise constant ansatz and test functions and consider standard adaptive algorithms of the type

Solve  $\longrightarrow$  Estimate  $\longrightarrow$  Mark  $\longrightarrow$  Refine

It is a simple consequence of functional analysis that the sequence  $u_\ell$  of Galerkin solutions generated by this algorithm, tends to some limit  $u_\infty \in \tilde{H}^{-1/2}(\Gamma)$ . It is, however, a priori unknown whether  $u_\infty$  coincides with the unique exact solution  $u \in \tilde{H}^{-1/2}(\Gamma)$  of the integral equation.

For a posteriori error estimation, we use certain  $(h - h/2)$ -type error estimators  $\mu_\ell$  from [4], and element marking is done by the  $\ell_2$ -criterion introduced by Dörfler [2]. We then treat the convergence

$$\lim_{\ell \rightarrow \infty} u_\ell = u \quad \text{as well as} \quad \lim_{\ell \rightarrow \infty} \mu_\ell = 0$$

for both, isotropic and anisotropic mesh-refinement.

### References

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3. S. Ferraz–Leite, C. Ortner, D. Praetorius: Convergence of simple adaptive Galerkin schemes based on  $h-h/2$  error estimators. Submitted for publication, 2009.
4. S. Ferraz–Leite, D. Praetorius: Simple a posteriori error estimators for the  $h$ -version of the boundary element method. Computing 83 (2008) 135–162.

## BEM-based FEM for eddy current problems

M. Fleck

Universität des Saarlandes, Saarbrücken

We analyse a method related to Domain Decomposition Methods and Trefftz-FEM. A Boundary Element Method is used to construct trial functions for Finite Element Methods on arbitrary polyhedral meshes. The functions are determined by their Dirichlet values on the boundaries of mesh elements.

While the choice of Dirichlet data for trial functions of polynomial degree is natural in the case of 2-dimensional scalar-valued problems, treatment of 3D vector-valued equations is more complicated. We go into the special case of eddy current problems and discuss strategies for constructing suited trial functions.

### References

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2. D. Copeland: Boundary–element–based finite element methods for Helmholtz and Maxwell equations on general polyhedral meshes. Technical Report 11, RICAM, 2008.
3. R. Hiptmair: Boundary Element Methods for Eddy Current Computation. In *Computational electromagnetics (Kiel, 2001)*, volume 28 of *Lect. Notes Comput. Sci. Eng.*, pp. 103–126, Springer, Berlin, 2003.

## Adaptive BEM for Mixed Boundary Value Problems

M. Aurada, P. Goldenits, M. Karkulik, D. Praetorius  
TU Wien

In our talk, we consider an adaptive BE scheme for the equivalent integral formulation of the Laplace equation in 2D with mixed boundary conditions. In the proposed scheme, the given boundary data and the non-homogeneous volume force are appropriately approximated by piecewise polynomials. Besides the possible singularities of the (in general unknown) solution, the adaptive mesh-refinement aims at a sufficient resolution of the data. We prove that the adaptive algorithm drives an extended estimator quantity, given as sum of an  $(h - h/2)$ -type error estimator and data oscillations, to zero. Under certain assumptions, this implies that the sequence of (computed) discrete solutions, in fact, tends to the (unknown) exact solution.

### References

1. M. Aurada, S. Ferraz–Leite, D. Praetorius: Estimator reduction and convergence of adaptive FEM and BEM, submitted for publication (2009).
2. M. Aurada, P. Goldenits, M. Karkulik, D. Praetorius: Adaptive BEM for some mixed boundary value problem, work in progress (2009).
3. G. Of, O. Steinbach, P. Urthaler: Fast evaluation of Newton potentials in the boundary element method, Institute for Numerical Mathematics, Graz University of Technology, 2008.

## **The Rankine boundary element method for the calculation of the potential flow around ships**

A. von Graefe

Germanischer Lloyd, Hamburg

GL Rankine is a seakeeping code for the calculation of ship motions and loads. It is a potential method using the Rankine boundary element method. Mainly, it consists of two parts: The calculation of the stationary and periodical flow around the ship. In both cases, water is assumed to be inviscid, incompressible and non rotational. Therefore, the Laplace equation with the corresponding boundary conditions has to be solved. The free water surface as well as the wet ship hull is discretized by a panel mesh.

In the stationary part, the ship moves forward steadily in plain water. The free surface is treated 'fully nonlinearly'. Hence, an iterative method is necessary. The waterline of the ship changes in every iteration step. Correspondingly, the panel mesh has to be adapted during the iteration.

Based on the stationary solution, a perturbation formulation is used in the periodical part. Here, the ships motions are caused by a harmonic wave with a given direction and encountering frequency. The ship is treated as a rigid body with six degrees of freedom. The boundary conditions are linearized in the periodical part. Therefore, the problem can be described and efficiently solved in the frequency domain.

## Boundary Element based Trefftz Methods for Potential Problems

C. Hofreither

Johannes Kepler Universität Linz

We present a Trefftz method employing locally harmonic ansatz functions for the solution of potential equations in two- or three-dimensional domains.

The method supports heterogeneous meshes consisting of various non-standard polygonal/polyhedral element shapes, as well as grids with hanging nodes. In the special case of a conforming triangular (in 2-D) or tetrahedral (in 3-D) mesh, the method is equivalent to the corresponding nodal finite element method with piecewise linear and continuous ansatz functions.

Using element-local Steklov-Poincaré operators, the formulation is reduced. Boundary element discretization is then employed in order to obtain a numerical scheme. We give error estimates and present first numerical results.

## Parallelization strategies in CASOPT

T. Klug

ABB Schweiz, Baden

One of the main objectives in the EU project CASOPT (Controlled Component- and Assembly Level Optimization of Industrial Devices) is to establish an automated optimization-based design process for electromagnetically-driven industrial products. Up to now, the POLOPT simulation software is used within ABB as a BEM solver which utilizes different fast methods like Fast Multipole or Adaptive Cross Approximation. In order to achieve an acceptable performance for multiple optimization runs, these methods have to be parallelized. This talk gives an introduction to parallel computer architectures and parallel programming paradigms. The basic design of distributed and shared memory machines is explained and appropriate programming models are covered. Latest developments with respect to Multi- and Many-core architectures are presented.



## **A BEM model of a tunnel in a layered orthotropic medium**

W. Kreuzer, G. Rieckh, H. Waubke

Austrian Academy of Sciences, Acoustics Research Institute, Wien

When using the boundary element method in combination with a layered anisotropic medium, one of the main problems is the lack of a usable closed form of the Green's function for such a medium. In this talk, we present a method to numerically calculate the fundamental solution on a given grid. This method is based on Fourier transforming the whole system with respect to time as well as space. To avoid numerical problems the BIE is solved in the Fourier domain. As an example we present a 3D-model of a tunnel in a layered orthotropic medium.

## A MHD problem on unbounded domains: Coupling of FEM and BEM

W. Lemster, G. Lube  
Universität Göttingen

We consider the magnetohydrodynamic (MHD) problem:

$$\begin{aligned}\partial_t \mathbf{B} &= -\nabla \times \mathbf{E} && \text{in } \Omega_c, \\ \nabla \times \frac{1}{\mu} \mathbf{B} &= \begin{cases} \sigma(\mathbf{E} + \mathbf{u} \times \mathbf{B} + j^c) & \text{in } \Omega_c, \\ 0 & \text{in } \Omega_v, \end{cases} \\ \nabla \cdot \mathbf{B} &= 0 && \text{in } \Omega.\end{aligned}$$

In the so-called direct problem, the magnetic induction  $\mathbf{B}$  and the electric field  $\mathbf{E}$  are unknown and  $\mathbf{u}$  is a given incompressible flow field. The domain  $\Omega$  consists of conducting regions  $\Omega$  and insulating regions  $\Omega_E$ . We apply a finite element approach (FEM) in  $\Omega$ . A boundary element approach (BEM) is used in  $\Omega_E$ . We use a symmetric coupling of both methods. We present results on the well-posedness of the continuous problem.

## **A general purpose Fast BEM library for the solution of time domain elastodynamic problems**

Ma. Messner, Mi. Messner, F. Rammerstorfer, P. Urthaler  
TU Graz

Wave propagation phenomena in reality occur often in semi-infinite regions. It is well known that such problems can be handled well with the Boundary Element Method (BEM). However, it is also known that the BEM, with its dense matrices, becomes prohibitive with respect to storage and computing time. The present work focuses on time dependent elastic problems. Their solution is speed up by using a BEM approach that is based on the ACA. This is enabled by introducing the Convolution Quadrature Method (CQM) as time stepping scheme. Thus the solution of time dependent problems ends up in the solution of a system of decoupled Laplace domain problems. This detour is worth since the resulting problems are elliptic and the ACA can be used in its standard fashion. The main advantage of this approach in accelerating a time dependent BEM is that it can be easily applied to other fundamental solution as, e.g., visco- or poroelasticity.

When dealing with wave propagation phenomena the computational effort does not only increase with the size of the boundary (for the present approach  $O(n \log n)$ ), but also linearly in time. This motivates the development of efficient solvers and equally important their efficient implementation. Instead of writing one single program that can handle anything, our aim is to implement low level, general purpose, and completely "orthogonal" modules. This approach results in a library structure and allows the design of specific high level solvers. In this sense, different programs, which lack unnecessary control instructions, can be build in order to solve specific problems. This library is written in C++ and is heavily templated. Hierarchical Matrices and their algebra are included since they can be used in a black box manner.

## **BEM/FEM coupling for transient Maxwell problems**

G. Of, O. Steinbach, S. Zaglmayr  
TU Graz

For the discretization of the transient Maxwell equations we consider the coupling of finite and boundary element methods. In particular, we will focus on hp finite element methods and a fast multipole approach. We will discuss several formulations to handle general smooth interfaces.

## Explicit constants for some boundary integral operators

C. Pechstein

Johannes Kepler Universität Linz

Among the well-known constants in the theory of boundary integral equations are the ellipticity constants of the single layer potential and the hypersingular boundary integral operator, and the contraction constant of the double layer potential. Whereas there have been rigorous studies how these constants depend on the size and aspect ratio of the domain, only little is known on their dependency on the shape of the boundary.

In this talk, we consider the homogeneous Laplace equation and derive explicit estimates for the above mentioned constants. It turns out that using an alternative trace norm, the dependency can be made explicit in two geometric parameters, the so-called Jones parameter and the constant in an isoperimetric inequality. There are many domains with quite irregular, ragged boundaries, where these parameters stay bounded.

## **ASBEST — Adaptive Symmetric Boundary Element Simulation Tool**

M. Aurada, M. Ebner, S. Ferraz–Leite, P. Goldenits, M. Karkulik,  
M. Mayr, D. Praetorius  
TU Wien

Currently, we develop a MATLAB library ASBEST for the lowest-order adaptive boundary element method for use in academic teaching and research. The library includes functions for the assembly of the Galerkin matrices for the integral operators associated with the 2D Laplacian as well as certain error estimators and certain adaptive mesh-refining strategies. ASBEST realizes the outcome of our recent research and makes it available to other researchers working in the field.

In our talk, we give a short overview on the current stage of ASBEST, which includes the integral formulations of the Laplace equation with Dirichlet and/or Neumann boundary conditions and with/without volume forces.

### **References**

1. Library and documentation are available online at <http://www.asc.tuwien.ac.at/abem/>

# Spectral BIE methods for Engineering Moonlit Night at Nanoscale<sup>1</sup>

L. Raguin, D. Bowler, C. Hafner, R. Vahldieck, R. Hiptmair  
ETH Zürich

Light is a powerful force affecting many aspects of natural and material life on the planet earth. Light can even bring nano-sized noble-metal particles to life, causing strong resonant oscillations of their conduction band electrons known as *plasmon resonance* [1]. The effect is so promising that plasmonic nanostructures have become the basis for novel applications in surface-enhanced Raman spectroscopy, chemical and biological sensing, and imaging. When looking for innovative nano-engineering ideas why not follow the tradition of being inspired by nature [2] and try to reconnect with the sky by taking a wide range of shapes and complex structural motifs from the starry, moonlit heavens as templates? Noble-metal moonlit nights at nanoscale have already been engineered by use of an isolated silver crescent nano-moon structure [3], resembling the real one but shining in seemingly infinite darkness as if the sky has been emptied of stars, planets and galaxies. Various shape modifications were considered, the real shape was forgotten in favour of creating man-made nano-moons featuring the desired properties but at any time of the day or night. Stars taken from sky, leaving behind a vacant haze that mirrors human fear of the darkness and the unknown, give rise to uniquely-shaped gold nanoparticles to be used in a range of applications from disease diagnostics through to the identification of contraband [4]. Planets, taken as templates for tiny spheres of silica, coated with a thin layer of noble-metal, known also as *nanoshells*, offer an efficient approach not only to detect cancer cells but also to destroy them. Coupling multiple nanoparticles in chain-like structures, resembling the Milky Way may be an approach to nanoscale optical waveguiding [5]. Galaxies of nanostars and nanoshells gathered together may form *hot spots* with a field enhancement dramatically larger than that for a single noble-metal nanoparticle. Such *hot spots* may be used to attract molecules just like insects clustering around streetlights. The investigation of noble-metal nanoparticles, their dimmers and clusters, using the ideas taken a velvet night are the subject of this work. Therefore, for all configurations considered in this work exact electro-dynamical calculations of plasmonic properties are based on the Boundary Integral Equation (BIE) method in combination with spectral Fourier discretization and regularization by means of Mie solution to reveal all of nature's secrets.

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<sup>1</sup>This work is supported financially by Swiss National Science Foundation project no. 200021-119976 "Spectral Galerkin Boundary Integral Equation methods for plasmonic nanostructures".

## The Use of Frames in BEM

G. Rieckh

Austrian Academy of Sciences, Acoustics Research Institute, Wien

Although one might associate redundancy and low compression rate with the concept of frames, it has several advantages over using basis. Frames are generally easier to construct and, if chosen in a way that fits the problem, can add to the stability of the method, and promote sparsity.

In our talk we present how the use of frames, rather than basis, can be advantageous when combined with the boundary element method, especially for the numerical solution of the Helmholtz equation. By using an overlapping decomposition of the respective domain, we obtain a frame for the solution space.

The solutions techniques for this kind of problems will also be looked into.

### References

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2. R. Stevenson: Adaptive solution of operator equations using wavelet frames, *SIAM J. Numer. Anal.* 41 (2003) 1074–1100.



## A variational integral formulation for fracture mechanics

A. Salvadori

Universita di Brescia

The fracturing process reveals three distinct phases [1]: loading without crack growth, stable crack growth, and unstable crack growth. During crack advancing, energy dissipation takes place in the process region, in the plastic region outside the process region, and eventually in the wake of the plastic region. When the fracture process is idealized to infinitesimally small scale yielding, energy dissipation during crack growth is concentrated at the crack tip.

For linear elastic fracture mechanics, the crack propagation has been studied in [2] exploiting its analogy with plasticity theory. A maximum principle was stated, that expressed the maximum dissipation at the crack tip during propagation; from it, associated flow rule and a propagation criteria for angle determination descend. Consistency conditions led to the formulation of an algorithm for crack advancing, which was driven by the increment of external actions (under the simplifying assumption of proportional loading) and allowed the evaluation of crack length increment and curvature at the crack tips of several cracks contemporarily advancing. This idea is here further pursued, by noting that Amestoy–Leblond [3] asymptotic expansion has an *effect superposition interpretation* thus allowing a Colonnetti’s approach. As a consequence, a minimum variational formulation is obtained in terms of crack tip velocity. It reminds to Ceradini’s theorem for plasticity.

Stability of crack path [4] is discussed as well. By the analogy with plasticity theory, general conditions for stability are stated for a general mixed mode crack growth, showing the role of the T stress in crack advancing.

Accurate evaluation of SIFs and T stress is achieved by means of boundary integral equations. Actual features and future developments are presented.

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## **Boundary element methods in dielectric media**

G. Of, O. Steinbach, P. Urthaler  
TU Graz

The modelling of electric fields in dielectric media results in a transmission boundary value problem for the potential equation with piecewise constant coefficients. A well established approach is based on an indirect single layer potential ansatz which results in a global system of second kind boundary integral equations. In addition we also describe a boundary element domain decomposition approach. We will discuss pros and cons of both approaches, and we will present first numerical results for a comparison of both methods.

## Stable coupling of finite and boundary elements

O. Steinbach  
TU Graz

We prove in the case of a Lipschitz interface the stability of the coupling of finite and boundary element methods when the direct boundary integral equation with single and double layer potentials is used only. In particular we prove an ellipticity estimate of the coupled bilinear form. Hence we can use standard arguments to derive stability and error estimates for the Galerkin discretization for all pairs of finite and boundary element trial spaces.

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1. O. Steinbach: A note on the stable coupling of finite and boundary elements. Berichte aus dem Institut für Numerische Mathematik, Bericht 2009/4, TU Graz, 2009.

## **Non-linear stress analyzes of cracked structures by the boundary element method**

W. Weber, K. Willner, P. Steinmann, G. Kuhn

Universität Erlangen–Nürnberg

For the assessment of the structural integrity of cracked structures numerical stress analyzes have to be carried out. On the one hand, the state of stress and strain assuming linear-elastic material behavior is determined for the simulation of fatigue crack propagation. On the other hand, elastic-plastic stress analyzes of the cracked structures are used for an assessment of the grown crack configurations according to established procedures, e.g. the failure assessment diagram.

The boundary element method (BEM) has been proven as a powerful tool for stress concentration problems like the present crack problem. In this talk the coincident crack surfaces are separated by the dual discontinuity method (DDM). The boundary integral equations are evaluated in the framework of a collocation procedure. Since the discontinuities of the displacements and tractions are utilized as primary variables at the crack the interaction of the crack surfaces are taken into account. The hard contact is softened by the penalty-method. A radial-return mapping scheme is applied for the solution of the frictional contact problem.

Although a domain discretization is needed in case of plasticity, the BEM is not losing its advantage of a reduced complexity concerning the mesh generation. As local plasticity has mainly to be considered in the crack near field, only this area has to be meshed with volume cells. Here, the talk is focused on the regularization process and the numerical evaluation of the domain integrals.

Several numerical examples are presented to demonstrate the influence of friction and plasticity on the stability of structures containing cracks.

# Adaptive FEM with local Trefftz trial functions for elliptic equations

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We discuss a special finite element method that solves the stationary isotropic heat equation with Dirichlet boundary conditions on arbitrary polygonal and polyhedral meshes. The method uses a space of locally harmonic trial functions to approximate the solution of the boundary value problem. According to this choice, we obtain a variational formulation on the skeleton of the domain. This formulation contains one Steklov-Poincaré-Operator for each element. These operators are constructed by means of boundary integral formulation. Therefore, the proposed finite element method can be used on general polygonal non-conform meshes. Hanging nodes are treated quite naturally. The material properties are assumed to be constant on each element. We also discuss adaptive mesh refinement to handle cases, when the material properties are given as a continuous function.

In a second step we have a look at a posteriori error estimates which can be used for further mesh refinement. Standard methods are based on triangular or quadrilateral meshes. The challenging part is to handle the arbitrary polygonal and polyhedral meshes. Therefore, we make use of functional analytic estimates to overcome these problems.

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## Preconditioned BETI for Helmholtz

O. Steinbach, M. Windisch

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In this talk we will present a boundary element tearing and interconnecting approach for the Helmholtz equation. In contrary to the Laplace equation it is in general not known if local Dirichlet or Neumann problems admit a unique solution. So one has to stabilize the standard approach to get rid of artificial eigenfrequencies of the local problems.

So we will present a stabilized approach which leads to a uniquely solvable discrete system. This will be done in two steps: First Robin boundary conditions are introduced to ensure the solvability of the local problem. But the Steklov-Poincare operator, which is used in the formulation may not well defined if the local Dirichlet problem is not uniquely solvable. So we introduce an alternative formulation for the local problem which leads to an always well defined and uniquely solvable formulation. Additionally, one can prove that also the discrete local and the discrete global problem have a unique solution. Then we present a preconditioning technique which was introduced by Farhat for Helmholtz-FETI, and finally we give some numerical examples.

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