Technische Universität Graz

17. Workshop on

Fast Boundary Element Methods in Industrial Applications

Söllerhaus, 3.–6.10.2019

U. Langer, M. Schanz, O. Steinbach, W. L. Wendland (eds.)

Berichte aus dem Institut für Angewandte Mathematik

Book of Abstracts 2019/7

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Technische Universität Graz
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Program

Thursday, October 3, 2019	
15.00	Coffee
16.25–16.30	Opening
16.30–17.00	J. Stocek (Edinburgh) Optimal operator preconditioning for pseudodifferential boundary problems on adaptive meshes
17.00–17.30	M. Ruggeri (Wien) The saturation assumption yields optimal convergence of two–level adaptive BEM
17.30–18.00	M. Bauer (Bayreuth) The proof of the cross approximation using interpolation by radial basis functions
18.00–18.30	D. Seibel (Saarbrücken) Fast boundary element methods for the wave equation
18.30	Dinner
Friday, October 4, 2019	
8.00–9.00	Breakfast
9.00–9.30	R. Hiptmair (Zürich) First–kind Galerkin boundary element methods for the Hodge–Laplacian in three dimensions
9.30–10.00	E. Schulz (Zürich) Coupled domain–boundary variational formulations for Hodge–Helmholtz operators
10.00–10.30	M. Kirchhart (Aachen) Vector potentials in bounded domains
10.30–11.00	Coffee
11.00–11.30	S. Kurz (Darmstadt/Stuttgart) Bembel, the boundary element method based engineering library
11.30–12.00	I. Chollet (Paris) Exploiting symmetries in multipole methods: cubature rules in HF–FMM
12.00–12.30	R. Watschinger (Graz) A parabolic FMM for the heat equation with non–uniform time steps
12.30	Lunch
14.30–15.00	U. Langer (Linz) Space–time finite element methods for parabolic initial–boundary value problems with low–regularity solutions
15.00–15.30	J. Hauser (Graz) Space–time finite element methods for Maxwell's equations
15.30–16.00	Coffee
16.00–16.30	G. Unger (Graz) Boundary integral formulations of eigenvalue problems for elliptic differential operators with singular interactions and their numerical approximation by boundary element methods
16.30–17.00	H. Yang (Linz) A space–time finite element method for optimal control of parabolic equations
17.00–17.15	Break
17.15–17.45	M. Feist (Bayreuth) Fractional Laplacian – Approximation of the dense FEM stiffness matrix by uniform $\mathcal{H}$–matrices
17.45–18.15	C. Erath (Darmstadt) Parabolic–elliptic interface problem on an unbounded domain: full discretization with the method of lines
18.30	Dinner

Saturday, October 5, 2019	
8.00–9.00	Breakfast
9.00–9.30	X. Claeys (Paris) A convergent optimised Schwarz method in arbitrary non–overlapping sub–domain partitions
9.30–10.00	H. Gimperlein (Edinburgh) Pseudodifferential equations in polygonal domains: Regularity and numerical approximation
10.00–10.30	D. Sebastian (Wien) Functional a posteriori error estimates for boundary element methods
10.30–11.00	Coffee
11.00–11.30	M. Aversang (Palaiseau) New preconditioners for the Helmholtz integral equation on screens
11.30–12.00	M. Elasmi (Darmstadt) Computation of forces in electro–mechanical energy converters using Isogeometric FEM–BEM coupling
12.00–12.30	P. Panchal (Zürich) Force computation using shape calculus
12.30	Lunch
13.30–18.00	Hiking Tour
18.30	Dinner
Sunday, October 6, 2019	
8.00–9.00	Breakfast
9.00–9.30	C. Urzua–Torres (Oxford) A new approach to space–time boundary integral equations for the wave equation
9.30–10.00	O. Steinbach (Graz) Finite and boundary element methods in thermoelasticity
10.00–10.30	Coffee

18. Söllerhaus Workshop

Fast boundary element methods in industrial applications

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New preconditioners for the Helmholtz integral equation on screens

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The standard Galerkin method for wave scattering problems by screens has a slow convergence rate in terms of the mesh–size, and the usual preconditioning techniques prove inefficient, due to the singularity of the domain itself. For the Laplace problem ($k = 0$), exact inverses of the classical single– and hypersingular layer potentials are well–known in 2D and 3D, [1, 2]. For small k, they can be exploited for preconditioning the integral equations. For large k, it becomes desirable to include a dependence in k in the preconditioner to preserve the performance of the method. An example of such an approach is that of Bruno and Lintner, who use weighted versions of the layer potentials as mutual preconditioners, with very good numerical results [3].
Here, we introduce alternative k–dependent preconditioners that do not rely on Calderon relations but rather on pseudo-differential analysis. Essentially, it consists in a generalization of the approximations of the Dirichlet–to–Neumann map proposed by Antoine and Darbas [4]. In our context, those approximations include a weight function that accounts for the singularity of the domain. One advantage over the method of Bruno and Antoine is the fact that preconditioners in the form of square–roots can be discretized efficiently.
In 2D, the introduction of the preconditioners is supported by a theoretical analysis relying on pseudo-differential operators on open curves. We show that the frequency correction they include with respect to the Laplace preconditioners is the one that exactly cancels the leading term in the Taylor approximation of the symbols of the layer potentials. Our pseudo–differential analysis on open curves is connected to a work of Saranen and Vainikko [5]. Numerical results, demonstrating the efficiency of the method, will be presented.

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The proof of the cross approximation using interpolation by radial basis functions

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The adaptive cross approximation (ACA) is a well known method for approximating functions and discrete non-local operators. The error of the approximation can be estimated by the best approximation error in any system of functions. The existing proof relies on the approximation with polynomials, which has some disadvantages, such as the additional requirement of the unique solvability of the interpolation problem or rather the correct choice of interpolation points.
The new approach is based on positive definite functions inducing a natural function space. Such spaces are typically known in the context of interpolating with radial basis functions (RBF). Here, the error of the approximation is estimated in terms of the fill distance. Accordingly, the correct choice of interpolation points is reduced to the minimization of the fill distance in the considered domain. Since RBFs are positive definite, the underlying interpolation problem can be solved uniquely without additional conditions.
Using the example of the singularity function of the Laplace operator in three dimensions, a positive definite kernel function is constructed which creates a natural function space.

Exploiting symmetries in multipole methods: cubature rules in HF–FMM

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Certain variants of the Fast Multipole Methods (FMM) naturally feature multiple symmetries, even when the geometry of the considered problem does not admit itself any particular symmetry. This leads to practical numerical optimizations. Such symmetries can be studied from the group theoretic viewpoint, which allows to take full computational advantage of them. We consider the case of interpolation between cubature grids in High Frequency FMM (HF–FMM), where certain matrix/vector products with particular cubature rules are accelerated using block-diagonalizations entirely given by group theory. This choice of particular non–tensorized cubature rule is justified by the size of the involved grids, and leads to a quasi–optimal complexity in one of the main steps of HF–FMM.

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A convergent optimised Schwarz method in arbitrary non–overlapping sub–domain partitions

X. Claeys

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The Optimized Schwarz Method (OSM) is a well established domain decomposition (DDM) strategy for solving frequency domain wave propagation problems such as Helmholtz equation [2, 4]. In this method, the wave equation is solved independently in each subdomain imposing impedance conditions at the boundary. Coupling between subdomains is obtained via an exchange operator that swaps traces on each side of each interface. Whenever the subdomain partition does not involve any junction i.e. point where at least three subdomains abut, this strategy can be very efficient provided that the impedance of local subproblems is chosen wisely [3].
The situation is different when there are junctions and the presence of such points can spoil the consistency of the method, even for common geometric configurations [1]. The treatment of junctions in OSM has been the subject of many contributions and, although convincing numerical remedies are now available in the case of right–angled junctions, no generic satisfactory approach has been proposed so far.
In this talk we will present a new variant of OSM where the exchange operator is defined through layer potentials and appears as a good candidate for dealing with junctions. We shall discuss in detail the properties of the operator associated to this new method.

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Computation of forces in electro–mechanical energy converters using Isogeometric FEM–BEM coupling

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[2]Centre for Computational Engineering, TU Darmstadt
[3]Departement of Mathematics, TU Darmstadt

Electro-mechanical energy converters are typically characterized by a moving and a stationary part, separated by a thin air gap. The movement is driven by electromagnetic forces caused by the interaction of the fields in the air gap. The computation of forces is therefore a central task in the simulation of these devices. Nevertheless, it is also challenging, since it is very sensitive to numerical errors. In order to reduce them, we use Non-Uniform Rational B-Splines (NURBS) for an exact representation of the domains. In addition, we consider a non-symmetric FEM-BEM coupling. Hence, we profit from the advantages of FEM for possibly non-linear and non-homogeneous domains, and of BEM for the unbounded and/or thin linear ones (e.g. air gap). In particular, NURBS parametrizations, in the FEM-BEM framework, facilitate the incorporation of movements and deformations. The discrete setting is then obtained by considering conforming B-Spline spaces, which offer higher order Ansatz functions, and thus, a higher regularity of the solution. Some numerical examples illustrate the theoretical analysis. Thereby, forces and torques are computed by means of the Maxwell stress tensor method.

Acknowledgment: The work of Mehdi Elasmi is supported by the 'Excellence Initiative' of the German Federal and State Governments and the Graduate School of Computational Engineering at TU Darmstadt.

Parabolic-elliptic interface problem on an unbounded domain: full discretization with the method of lines

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In electromagnetism, the quasistatic approximation of a two dimensional eddy current problem ends up in a parabolic-elliptic interface problem [1]. Another application of such a interface problem arises in fluid mechanics. Hence, we have a special focus of different physical properties of the solution in different parts of the domain and on unbounded domains. Therefore it makes sense to consider couplings of different methods to get the best possible numerical approximation. The general strategy to get a fully discrete numerical method in our presentation is the method of lines approach.
First, we apply a non-symmetric FEM-BEM coupling [2] to discretize the spatial direction - semi discretization. For the subsequent time discretization we choose a standard ODE discretization scheme. We aim to provide the first complete numerical analysis of such a coupling for the fully discrete system which also holds on Lipschitz domains [3]. Even more, if we apply a variant of the backward Euler scheme for time discretization we can prove quasi-optimality under minimal regularity assumptions also for the fully discrete system. Note that the bottleneck in the analysis is the lack of adjoint consistency. Our estimates are done in appropriate energy norms and do not rely on duality arguments. Moreover, we discuss the extension of our model problem to a problem arising in fluid mechanics. However, since the conservation of fluxes is mandatory for such applications, we replace the Finite Element Method by the Finite Volume Method in our coupling approach [4]. Numerical examples illustrate the predicted (optimal) convergence rates and underline the potential for practical applications.

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Fractional Laplacian – Approximation of the dense FEM stiffness matrix by uniform $\mathcal{H}$–matrices

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$\mathcal{H}$-matrices are a well suited method to approximate discrete elliptic operators. Here, we want to show that the fractional Laplacian $L^s = (-\Delta)^s$, $0 < s < 1$, can also be approximated by uniform $\mathcal{H}$-matrices. Although, the fractional Laplacian is an elliptic operator, there are some special features, which we have to consider, before we can start the approximation. The operator is non-local and its integral form is used to discretize it. Therefore, we have to handle a dense stiffness matrix. Additionally, each entry consists of combinations of volume and surface integrals, which make the computation quite costly. Thus, an efficient treatment of these entries is needed. We will use the similarity between the integral form of the fractional Laplacian and boundary integral equations to develop an efficient computation method. The far field blocks of the matrix will be approximated by uniform $\mathcal{H}$-matrices. For each cluster we compute one approximation being valid for all far field blocks which contain this cluster. In order to get a low rank approximation of a far field block only the approximation of the kernel is needed for each cluster. Therefore, most of the computations can be done before the matrix approximation, which reduces the computational costs.
Additionally, we want to adapt the ideas used for the far field to the near field, especially for the singular integrals and for the nearly singular integrals. We generate an uniform approximation of the kernel for each tetrahedron. Thereby, the $2d$ dimensional integrals are decomposed to d dimensional integrals which can be computed separately.

Pseudodifferential equations in polygonal domains: Regularity and numerical approximation

Heiko Gimperlein

Department of Mathematics, Heriot Watt University Edinburgh, UK

Boundary integral formulations of screen problems or problems with mixed boundary conditions are classical examples of pseudodifferential boundary or interface problems. Fractional Laplacians lead to further examples of recent interest. The solutions of such problems exhibit singular behavior at the boundary or interface, resulting in slow convergence of Galerkin approximations by an h-method on quasi-uniform meshes. We discuss a unified approach to the regularity of pseudodifferential problems at boundaries, corners and interfaces. For the Dirichlet problem in a domain with smooth boundary, our approach recovers recent results of G. Grubb, using independent techniques. As an application, we obtain quasi-optimal convergence rates for Galerkin approximations on graded meshes.
(joint work with N. Louca, R. Mazzeo, E.P. Stephan, J. Stocek)

Space–time finite element methods for Maxwell's equations

Julia Hauser, Olaf Steinbach
Institut für Angewandte Mathematik, TU Graz, Austria

We consider Maxwell's equations in a space-time setting and the corresponding variational formulations. In particular we examine the vectorial wave equation for the electric field E including the spatial curl operator. In order to derive a suitable variational formulation we apply integration by parts both in time and space. The appropriate functional spaces for unique solvability of the resulting Galerkin-Petrov formulations we discussed in [1].
If we consider full space-time numerical examples, a tensor product ansatz might first come to mind. By inserting the ansatz the integrals split into their time and spacial parts and we end up computing only the spatial and time system matrices, using the Kronecker product and adding them in a suitable way. However we pay for this simplicity with a CFL condition. This gives motivation to look for other suitable finite elements.
Taking a step back and considering the ansatz space $H_{0;}^{curl;1}(Q)$, which was described in [1], we take a look at a conform set of finite elements. For that we consider the case of two dimensions in space and one in time. We discuss what properties are needed and what kind of meshes we might consider. In the end we consider first numerical examples.

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First–kind Galerkin boundary element methods for the Hodge–Laplacian in three dimensions

Ralf Hiptmair

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Boundary value problems for the Euclidean Hodge-Laplacian in three dimensions $\boldsymbol{\Delta}_{\mathrm{HL}} := \mathbf{curl}\,\mathbf{curl} - \mathbf{grad}\,\mathrm{div}$ lead to variational formulations set in subspaces of $\boldsymbol{H}(\mathbf{curl},\Omega)\cap\boldsymbol{H}(\mathrm{div},\Omega)$, $\Omega\subset\mathbb{R}^3$ a bounded Lipschitz domain. Via a representation formula and Calderón identities we derive corresponding first-kind boundary integral equations set in trace spaces of $H^1(\Omega)$, $\boldsymbol{H}(\mathbf{curl},\Omega)$, and $\boldsymbol{H}(\mathrm{div},\Omega)$. They give rise to saddle-point variational formulations and feature kernels whose dimensions are linked to fundamental topological invariants of Ω.

Kernels of the same dimensions also arise for the linear systems generated by low-order conforming Galerkin boundary element (BE) discretization. On their complements, we can prove stability of the discretized problems, nevertheless. We prove that discretization does not affect the dimensions of the kernels and also illustrate this fact by numerical tests.

Vector potentials in bounded domains

Matthias Kirchhart
RWTH Aachen

We consider the following problem. Let the vorticity ω be given on a bounded domain $\Omega \subset \mathbb{R}^3$ which has kinks and corners, for example a polyhedron. Find the velocity u, such that $\nabla \times u = \omega$ and $\nabla \cdot u = 0$. This is a classical problem and on the whole-space $\mathbb{R}^3$ its solution is given by the Biot–Savart law. In bounded domains, however this problem turns out to be significantly more subtle. A common approach is to find a so-called vector potential Ψ, such that $u = \nabla \times \Psi$. However, all approaches known to us yield a potential Ψ that can actually be less smooth than u itself. In our opinion, this is very counter-intuitive. In this talk we will discuss some of the many pitfalls we have stepped into while investigating this problem and present an approach based on the boundary element method which we believe solves this problem and yields a smooth potential. Like the original problem, this solution seems somehow classical and somehow not. We kindly ask the audience for their comments and if they are aware of any references that we are not.

Bembel, the Boundary Element Method Based Engineering Library

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We will talk about the story of Bembel, the Boundary Element Method Based Engineering Library. Based on the isogeometric paradigm, this software suite was developed in cooperation with the groups of Prof. Harbrecht (Basel) and Prof. Multerer (Lugano). We will briefly review basic notions and, afterwards, show the code's capabilities through a series of numerical examples, passing by scalar problems, electromagnetic scattering, electromagnetic eigenvalue problems, as well as comparisons to Raviart-Thomas and spectral elements.
Reference: www.bembel.eu

Space-time finite element methods for parabolic initial–boundary value problems with low–regularity solutions

Ulrich Langer and Andreas Schafelner

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We consider locally stabilized, conforming finite element schemes on completely unstructured simplicial space-time meshes for the numerical solution of parabolic initial–boundary value problems with variable, possibly discontinuous in space and time coefficients. Discontinuous coefficients, non–smooth boundaries, changing boundary conditions, non–smooth or incompatible initial conditions, and non–smooth right–hand sides can lead to non-smooth solutions. For instance, in electromagnetics, permanent magnets cause line–delta–distributions in the source term in 2d quasi–magnetostatic simulations of electrical machines.

We present new a priori and a posteriori error estimates for low–regularity solutions. In order to avoid reduced convergence rates appearing in the case of uniform mesh refinement, we also consider adaptive refinement procedures based on residual a posteriori error indicators and functional a posteriori error estimators. The latter provides guaranteed upper bounds on the error. The huge system of space–time finite element equations is then solved by means of GMRES preconditioned by space-time algebraic multigrid. In particular, in the 4d space–time case that is 3d in space, simultaneous space–time parallelization can considerably reduce the computational time. We present and discuss numerical results for several examples possessing different regularity features. The implementation is performed within MFEM.

The authors would like to thank the Austrian Science Fund (FWF) for the financial support under the grant DK W1214–04.

Force computation using shape calculus

Piyush Panchal
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In the Electrostatics setting, it is often of interest to compute the local/global forces on a body in presence of an Electric field. The volume formula for calculating the force, being stable in the energy norm, is a good choice when working with Finite Element Methods. The boundary formula however faces stability issues, making it difficult to compute forces when working with Boundary Element Methods. In my thesis/presentation, I talk about a new boundary formula derived using shape calculus, which is stable in the energy norm and thus suitable for use with BEM. I also compare its performance with the conventional boundary formula.

The saturation assumption yields optimal convergence of two–level adaptive BEM

Michele Ruggeri

Institute for Analysis and Scientific Computing, TU Wien, Austria

We consider the convergence of adaptive BEM for weakly-singular and hypersingular integral equations associated with the Laplacian and the Helmholtz operator in 2D and 3D. The local mesh-refinement is driven by some two-level error estimator. We show that the adaptive algorithm drives the underlying error estimates to zero. Moreover, we prove that the saturation assumption implies linear convergence of the error with optimal algebraic rates. This is joint work with Dirk Praetorius (TU Wien) and Ernst P. Stephan (Leibniz University Hannover).

Functional a posteriori error estimates for boundary element methods

Stefan Kurz[1], Dirk Pauly[2], Dirk Praetorius[3], Sergey Repin[4,5], Daniel Sebastian[3]
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This work motivates a new perspective on a posteriori error estimation for boundary element methods. In contrast to the state of the art, we aim for *fully computable* lower and upper bounds of the energy error $\|\nabla(u-u_h)\|_{L^2(\Omega)}$, where the approximate solution u_h stems from an either lowest-order direct or indirect approach to solve the Dirichlet problem

$$\Delta u = 0 \text{ in } \Omega \subset \mathbb{R}^d, \quad u_{|\Gamma} = g \text{ on } \Gamma = \partial\Omega \tag{1}$$

via Symm's integral equation.
One major advantage of *functional-type* estimates is, that they do not depend on a priori knowledge of the approximation u_h. As an example, Galerkin-orthogonality is a widespread tool which is obviously not available for a BEM approximation u_h on Ω. BEM's distinguishing feature that the error $(u-u_h)$ solves (1) exactly is the only ingredient to conclude the sharp error identity

$$\max_{\substack{\boldsymbol{\tau}\in L^2(\Omega)\\ \mathsf{div}\boldsymbol{\tau}=0}} \left[2\left\langle (g-u_{h|\Gamma})\,,(\boldsymbol{n}\cdot\boldsymbol{\tau})_{|\Gamma}\right\rangle - \|\boldsymbol{\tau}\|^2_{L^2(\Omega)}\right] = \|\nabla(u-u_h)\|^2_{L^2(\Omega)} = \min_{\substack{w\in H^1(\Omega)\\ w_{|\Gamma}=g-u_{h|\Gamma}}} \|\nabla w\|^2_{L^2(\Omega)}$$

which leads to maximization/minimization procedures.
Instead of solving the related problems on Ω, we suggest a construction via FEM on an adaptively shrinking boundary layer $S \subset \Omega$. Our computations on S are used to steer an adaptive mesh-refinement on Γ to regain the optimal rate of convergence

$$\|\nabla(u-u_h)\|_{L^2(\Omega)} \le C_{\text{cont}}\|\phi-\phi_h\|_{H^{-1/2}(\Gamma)} = \mathcal{O}(N^{-3/2}).$$

On top of that, our numerical examples verify that the majorant provides a reasonable stopping criterion, in order to guarantee the integrity of the final approximate solution.

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Fast boundary element methods for the wave equation

Daniel Seibel

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Boundary Element Methods (BEM) have been successfully used in acoustics, optics and elastodynamics to solve transient problems numerically. However, the storage requirements are immense, since the fully populated system matrices have to be computed for a large number of time steps or frequencies. In this talk, we propose a new approximation scheme for the Convolution Quadrature Method (CQM) based BEM applied to the wave equation. We use $\mathcal{H}^2$-matrix compression in the spatial domain and employ a generalised adaptive cross approximation algorithm in the frequency domain. In this way, the storage and computational costs are reduced significantly, while the accuracy of the method is unchanged.

Finite and boundary element methods in theromoelasticity

Olaf Steinbach
Institut für Angewandte Mathematik, TU Graz, Austria

Although the numerical simulation of problems in thermoelasticity is well established in particular in the engineering community, it seems that there is no detailed numerical analysis available yet. As model problem we consider the stationary system of thermoelasticity, where the heat equation is decoupled from the linear elasticity problem with the temperature gradient as volume source. The most common approach is to consider the variational formulation of both equations in the energy space $H^1(\Omega)$ where we can apply standard arguments for the stability and error analysis of the numerical scheme. But it is sufficient, similar as for the Stokes system, to consider the temperature in $L^2(\Omega)$ only, i.e., we consider an ultra weak variational formulation for the heat equation. We provide a related numerical analysis showing optimal error estimates, and we give some numerical examples. Then we discuss the relevance of these results for the boundary element approximation of the thermoelasticity system. We recall the standard boundary integral formulation, and we discuss related error estimates. We finally comment on the time–dependent problem.
This talk is based on joint work with D. Pacheco and M. Schanz.

Optimal operator preconditioning for pseudodifferential boundary problems on adaptive meshes

Jakub Stocek

We propose an operator preconditioner for general elliptic pseudodifferential equations in a domain Ω, where Ω is either in $\mathbb{R}^n$ or in a Riemannian manifold. For linear systems of equations arising from low-order Galerkin discretizations, we obtain condition numbers that are independent of the mesh size and of the choice of bases for test and trial functions.

The basic ingredient is a classical formula by Boggio for the fractional Laplacian, which is extended analytically. In the special case of the weakly and hypersingular operators on a line segment or a screen, our approach gives a unified, independent proof for a series of recent results by Hiptmair, Jerez-Hanckes, Nédélec and Urzúa-Torres.

We study the increasing relevance of the regularity assumptions on the mesh with the operator order. We discuss the impact of these assumptions on adaptively generated meshes. Numerical examples validate our theoretical findings and illustrate the performance of the proposed preconditioner on quasi-uniform, graded and adaptively generated meshes.

Boundary integral formulations of eigenvalue problems for elliptic differential operators with singular interactions and their numerical approximation by boundary element methods

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In this talk the discrete eigenvalues of elliptic second order differential operators in $L^2(\mathbb{R}^n)$, $n \in \mathbb{N}$, with singular δ- and δ'-interactions are analyzed. We show the self-adjointness of these operators and derive equivalent formulations for the eigenvalue problems involving boundary integral operators. These formulations are suitable for the numerical computations of the discrete eigenvalues and the corresponding eigenfunctions by boundary element methods. We provide convergence results and show numerical examples.

References

[1] M. Holzmann, G. Unger: Boundary integral formulations of eigenvalue problems for elliptic differential operators with singular interactions and their numerical approximation by boundary element methods. Berichte aus dem Institut für Angewandte Mathematik, 19/5, TU Graz, 2019.

A new approach to space–time boundary integral equations for the wave equation

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Space–time discretization methods are becoming increasingly popular, since they allow adaptivity in space and time simultaneously, and can use parallel iterative solution strategies for time–dependent problems. However, in order to exploit these advantages, one needs to have a complete numerical analysis of the corresponding Galerkin methods. Different strategies have been used to derive variational methods for the time domain boundary integral equations for the wave equation. The more established and succesful ones include weak formulations based on the Laplace transform, and also time–space energetic variational formulations. However, their corresponding numerical analyses are still incomplete and present difficulties that are hard to overcome, if possible at all. As an alternative, we pursue a new approach to formulate the boundary integral equations for the wave equation, which aims to provide the missing mathematical analysis for space–time boundary element methods. In this talk, we discuss some of our preliminary results.

A causal FMM for a space–time BEM for the heat equation with non–uniform time steps

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In this talk we consider a space-time boundary element method for the solution of the heat equation. In space-time methods the temporal component is regarded as an additional spatial component and the problem is solved in space and time as a whole. This increases the computational effort, but allows for parallelism and adaptivity in space and time. While tensor product meshes are widely used for the discretization of the space-time boundary, arbitrary decompositions of the boundary are more suitable when it comes to adaptivity.
As a first step towards general adaptive meshes, we focus on non-uniform time steps in this talk. For this purpose we consider a Dirichlet boundary value problem for the heat equation on the surface of the sphere as a model problem. If the Dirichlet datum is constant in space this can be reduced to a purely temporal problem. We discuss a fast method for its solution based on a clustering strategy and interpolation of the kernel. In particular we present adaptations of the fast method for non-uniform time steps. Finally, we demonstrate the advantages of the considered approach with some numerical examples.

A space-time finite element method for optimal control of parabolic equations

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In this talk, we will present some numerical methods for optimal control of parabolic PDEs. In particular, we aim to minimize certain objective functionals, that may involve a Lipschitz continuous and convex but not Fréchet differentiable term, subject to linear/nonlinear parabolic PDEs and under proper constraints on the control variables.
The space-time finite element discretization of the optimality system, including both the state and adjoint state equations, relies on a Galerkin–Petrov variational formulation employing piecewise linear finite elements on unstructured simplicial space-time meshes.
The nonlinear optimality systems of equations are solved by the semismooth Newton method, whereas the linearized coupled state and adjoint state systems are solved by an algebraic multigrid preconditioned GMRES method.
This is a joint work with Ulrich Langer (RICAM), Olaf Steinbach (TU Graz) and Fredi Tröltzsch (TU Berlin).

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