Technische Universität Graz

8. Workshop on

# Fast Boundary Element Methods in Industrial Applications

Söllerhaus, 30.9.–3.10.2010

U. Langer, O. Steinbach, W. L. Wendland (eds.)

**Berichte aus dem**
**Institut für Numerische Mathematik**

Book of Abstracts 20010/12

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## Berichte aus dem Institut für Numerische Mathematik

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Technische Universität Graz
Institut für Numerische Mathematik
Steyrergasse 30
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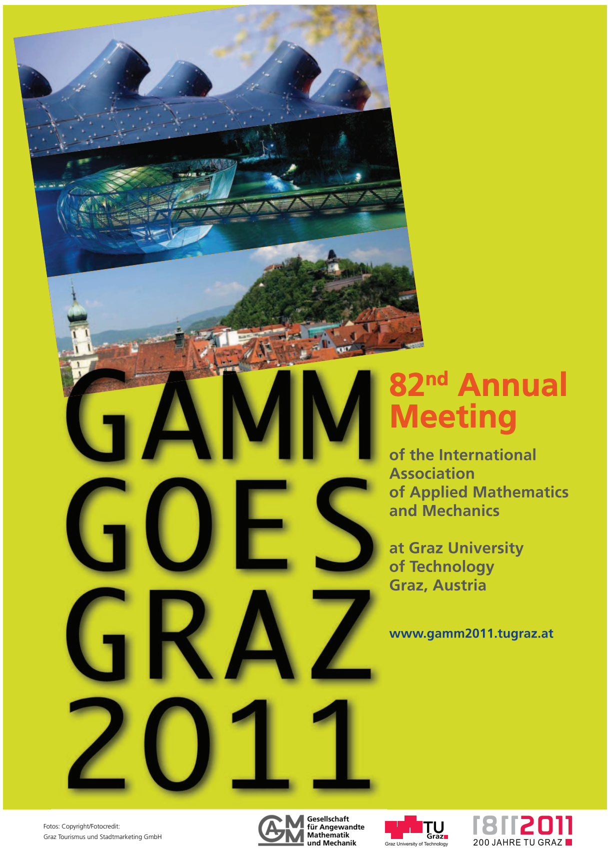
## Program

| Thursday, September 30, 2010 | |
|---|---|
| 15.00–16.20 | Coffee |
| 16.20–16.30 | Opening |
| 16.30–17.00 | S. Hardesty (Houston)<br>Optimization of shell structure acoustics |
| 17.00–17.30 | T. Betcke (Reading)<br>Coercivity and numerical range of boundary integral operators |
| 17.30–18.00 | D. Lukas (Ostrava)<br>Optimal TBETI for multi–body contact problems |
| 18.00–18.30 | G. Unger (Linz)<br>A boundary element method for Laplacian eigenvalue problems |
| 18.30 | Dinner |
| **Friday, October 1, 2010** | |
| 9.00–9.30 | E. P. Stephan (Hannover)<br>Fast solvers for the hp–version boundary element method and applications in electromagnetics |
| 9.30–10.00 | L. Weggler (Saarbrücken)<br>Stabilized boundary element formulation for Maxwell |
| 10.00–10.30 | S. Engleder (Graz)<br>Boundary element methods for low–frequency Maxwell problems |
| 10.30–11.00 | Coffee |
| 11.00–11.30 | C. Hofreither (Linz)<br>$L_2$ error estimates for a BEM–based FEM |
| 11.30–12.00 | S. Weisser (Saarbrücken)<br>Adaptive BEM based FEM on general polygonal meshes and residual error estimators |
| 12.00–12.30 | M. Feischl (Wien)<br>Convergence of some adaptive FEM–BEM coupling |
| 12.30 | Lunch |
| 15.00–15.30 | S. Kurz (Tampere)<br>Differential forms and boundary integral equations |
| 15.30–16.00 | M. Fleck (Saarbrücken)<br>Discrete electromagnetism of higher polynomial degree |
| 16.00–16.30 | Coffee |
| 16.30–17.00 | M. Betcke (London)<br>Image reconstruction for real time cone beam CT |
| 17.00–17.30 | S. Ferraz–Leite (Wien)<br>A quadratic minimization problem in thin–film micromagnetics: non–local, non–linear, and infinite dimensional |
| 17.30–18.00 | M. Windisch (Graz)<br>BETI methods for scattering problems |
| 18.00–18.30 | C. Jerez–Hanckes (Zürich)<br>Multiple traces boundary integral formulations for Helmholtz transmission problems |
| 18.30 | Dinner |

| Saturday, October 2, 2010 | |
|---|---|
| 9.00–9.30 | M. Schanz (Graz)<br>Time domain BEM: Brief overview and trends |
| 9.30–10.00 | L. Banjai (Leipzig)<br>Runge–Kutta convolution quadrature: New convergence results and applications to acoustic scattering |
| 10.00–10.30 | V. Gruhne (Leipzig)<br>Approximation of convolution weights for 2D and dissipative wave equation |
| 10.30–11.00 | Coffee |
| 11.00–11.30 | J. Zechner (Graz)<br>Application of hierarchical matrices to a BEM plasticity algoritm in tunneling |
| 11.30–12.00 | Ma. Messner (Graz)<br>A fast directional multilevel summation method for oscillatory kernels based on Chebyshev interpolation and adaptive cross approximation |
| 12.00–12.30 | P. Urthaler (Graz)<br>Comparison of boundary element methods for magnetostatic field problems |
| 12.30 | Lunch |
| 13.30–18.00 | Hiking Tour |
| 18.30 | Dinner |
| Sunday, October 3, 2010 | |
| 9.00–9.30 | M. Karkulik (Wien)<br>Application of interpolation theory to adaptive 3D–BEM |
| 9.30–10.00 | L. Raguin (Zürich)<br>Spectral Galerkin method for surface integral equations on nanoparticles in three dimensions |
| 10.00–10.30 | Coffee |
| 10.30–11.00 | G. Karlis, L. Malinowski (Graz)<br>Iterative coupling of DEM–BEM regions with an overlapping FEM zone |
| 11.00–11.30 | O. Steinbach (Graz)<br>Variational inequalities and boundary element methods |
| 11.30 | Closing |




April 18–21, 2011

**Runga-Kutta convolution quadrature: New convergence results and applications to acoustic scattering**

L. Banjai
MPI Leipzig, Germany

It is well-known that order reduction can occur when discretizing stiff differential systems by Runge-Kutta methods. This phenomenon for parabolic systems and corresponding sectorial convolution quadratures has been analysed by Lubich & Ostermann in 1993. We extend these results to the hyperbolic case, that is to operators whose Laplace transform is bounded polynomially in the right half complex plane.
Recently a need to refine the analysis for operators that are differently bounded in sectors of the right half-plane than in the whole half-plane, has been recognised in relation to applications coming from acoustics and electromagnetism. We present a different type of analysis for this class of problems and obtain a refined result that has some surprising consequences. Numerical experiments and applications coming from solving time-domain boundary integral equations of acoustic scattering will also be presented.
This is a joint work with Christian Lubich (Universität Tübingen) and Jens Markus Melenk (TU Wien).

**Image Reconstruction for Real Time Cone Beam CT**

M. M. Betcke
University College London, UK

In the quest of engineering a real time tomograph, the mechanical motion of the gantry was identified as a main bottle neck in increasing speed of measurements acquisition in state of the art cone beam scanners. Therefore in the new generation of the cone beam systems the mechanically rotating gantry was replaced by a stationary ring of sources, which can by quickly switched on and off by the on board electronics, and multiple stationary rings of detectors. To accommodate the stationary ring of sources in the design, it was necessary to divert from the 4th generation CT geometry. The resulting new geometry requires new different reconstruction algorithms than those devised for the standard cone beam CT. In this contribution we present a new family of methods, multi-sheet rebinning methods, generalising the rebinning methods to the new scanner geometry.

**Coercivity and numerical range of boundary integral operators**

T. Betcke
University of Reading, UK

Coercivity is an important concept for proving existence and uniqueness of solutions to variational problems in Hilbert spaces. But, while the existence of coercivity estimates is well known for many variational problems arising from partial differential equations, it is still an open problem in the context of boundary integral operators arising from acoustic scattering problems, where rigorous coercivity results have so far only been established for combined integral operators on the unit circle and sphere.

One way to interpret coercivity is by considering the numerical range of the operator. The numerical range is a well established tool in spectral theory and algorithms exist to approximate the numerical range of finite dimensional matrices. We can therefore use Galerkin projections of the boundary integral operators to approximate the numerical range of the original operator.

By computing the numerical range of the combined integral operator in acoustic scattering for exterior soundsoft scattering for several interesting convex, nonconvex, smooth and polygonal domains, we numerically study coercivity estimates for varying wavenumbers. Surprisingly, it turns out that for many domains a coercivity result seems to hold independently of the wavenumber or with only a mild wavenumber dependence on it. Also, there are very interesting connections to the nonnormality of the operator. In fact, using the example of a trapping domain we demonstrate that the loss of coercivity does not seem predictable purely from spectral information and that coercivity is strongly dependent on the distance to the nearest exterior resonance.

**Boundary Element Methods for low-frequency Maxwell Problems**

S. Engleder, O. Steinbach
TU Graz, Austria

In this talk we discuss boundary element formulations for the solution of boundary value and transmission problems arising from applications in electromagnetism. In particular we deal with applications in the low frequency range. We investigate the properties of boundary integral operators when dealing with low frequencies and present a new formulation, which is stable if we let the frequency tend to zero. Furthermore we give numerical examples to illustrate the theoretic results.

**Convergence of Some Adaptive FEM–BEM Coupling**

M. Feischl
TU Wien, Austria

We consider different FEM–BEM coupling methods for the numerical solution of some interface problem for the 2D Laplacian. Based on stability results from [1] and [2], we introduce some new a posteriori error estimators based on the (h-h/2)-error estimation strategy. In particular, these include the approximation error for the boundary data, which allows to work with discrete boundary integral operators only. Using the concept of estimator reduction, we prove that the proposed adaptive algorithm is convergent in the sense that it drives the underlying error estimator to zero. Numerical experiments underline the effectivity of the considered adaptive mesh-refinement and compare the (h-h/2)-approach with other adaptive strategies proposed in e.g. [1] and [3].

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**A quadratic minimization problem in thin-film micromagnetics: non-local, non-linear, and infinite dimensional**

S. Ferraz–Leite, J. M. Melenk, D. Praetorius
TU Wien, Austria

We consider the reduced model proposed in [3] which is consistent with the prior works [1] and [4] and is valid for sufficiently large and thin ferromagnetic samples. Let $\omega \subseteq \mathbb{R}^2$ denote a bounded Lipschitz domain with diameter $\ell \sim 1$. This domain represents our ferromagnetic sample $\Omega = \omega \times (0, t)$, whose thickness $t > 0$ is neglected for simplicity. Here, we consider a uniaxial material with in-plane easy axis $\mathbf{e}_1$. With an applied exterior field $\mathbf{f} : \omega \to \mathbb{R}^2$, we seek a minimizer $\mathbf{m}^*$ of the reduced energy

$$e(\mathbf{m}) = \frac{1}{2} \int_{\mathbb{R}^3} |\nabla u|^2 \, dx + \frac{q}{2} \int_{\omega} \mathbf{m}_2^2 \, dx - \int_{\omega} \mathbf{f} \cdot \mathbf{m} \, dx \tag{1}$$

under the convex side constraint $|\mathbf{m}| \leq 1$. The magnetostatic potential $u : \mathbb{R}^3 \to \mathbb{R}$ is related to the magnetization via

$$\int_{\mathbb{R}^3} \nabla u \cdot \nabla \varphi \, dx = \int_{\omega} \mathbf{m} \cdot \nabla \varphi(x, 0) \, dx \quad \text{for all} \quad \varphi \in \mathcal{D}(\mathbb{R}^3). \tag{2}$$

The representation of $u$ as simple-layer potential of $\nabla \cdot \mathbf{m}$, allows to rewrite the energy functional

$$e(\mathbf{m}) = \|\nabla \cdot \mathbf{m}\|_V^2 + \|\mathbf{m}_2\|_{L^2}^2 - (f, \mathbf{m})_{L^2},$$

with $\| \cdot \|_V$ denoting the non-local norm induced by the simple-layer potential associated with the Laplace operator in 3D. This observation leads to the choice of a certain subspace of $H^{1/2}(\mathrm{div}, \omega) := \{\mathbf{m} \in L^2(\omega)^2 \,|\, \nabla \cdot \mathbf{m} \in \widetilde{H}^{-1/2}(\omega)\}$ as energy space. Existence and uniqueness of a minimizer $\mathbf{m}^*$ in our functional setting is proven.

Based on some regularity results from [2], we propose a numerical discretization strategy by use of lowest-order Raviart-Thomas finite elements. Furthermore, algorithmic treatment of the side-constraint $|\mathbf{m}| \leq 1$ by use of a penalty method is analyzed. Convergence of our numerical scheme is studied, and numerical examples conclude the talk.

## References

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**Discrete electromagnetism of higher polynomial degree**

M. Fleck

Universität des Saarlandes, Germany

The basic principle of discrete electromagnetism (DEM) is to discretise the operators occuring in the differential form representation of a given PDE. While the proper spaces for versions of higher polynimial degree are known, there are different concepts for degrees of freedom and the corresponding shape forms. We compare two dissimilar concepts and analyse the discrete electromagnetism using a hierarchical set of higher order shape forms.

**Approximation of Convolution Weights for 2D and Dissipative Wave Equation**

V. Gruhne

MPI Leipzig, Germany

When solving hyperbolic partial differential equations in an exterior domain, first notably the wave equation as it appears in the field of propagation and scattering of acoustic or electromagnetic waves, it can be rewritten in a boundary integral equation which we solve with the aid of boundary element method. Here, the convolution quadrature approach of Lubich comes into play in order to discretize the integral in time domain. It makes use of convolution weights which in general are only given implicitly via a contour integral. If Huygens' principle holds, the forward tail of the convolution weights' kernels can be replaced by zero, so-called cut off strategy, and so storage and computational costs are reduced. In two space dimensions or when a dissipative term is involved, this approach is not realizable anymore, since in these cases Huygens' principle is not valid.

In this talk we show, that, nevertheless, one can extend the idea of cutting off to situations where Huygens' principle is violated. We discuss the possibility of approximating the convolution weight functions with the help of time domain's kernel function of the time-space boundary integral and use this result to justify interpolation of the convolution weights. We verify this approach with a numerical experiment.

**Optimization of Shell Structure Acoustics**

S. Hardesty
Rice University, Houston, USA

It is natural to model shell structure acoustics by coupling shell equations with boundary integral equations for the acoustics: the whole coupled three-dimensional problem can be elegantly formulated on a two-dimensional reference domain, removing the need for re-meshing during optimization so long as the design changes are not too large. The boundary integral equations yield solutions exactly satisfying the Sommerfeld radiation condition, without the need for artificial truncation of the exterior domain, and lend themselves to point-measurement of the external field via a representation formula.

While simpler structural acoustic optimization problems, e.g., the minimization of measured noise at a particular frequency, may be solved with sensitivities and a small shape parameter set, more interesting design problems may require large shape parameter sets. This is due both to the difficulty involved in the a priori choice of a smaller parameter set, and to the need to allow shape variations on a scale near that of the underlying finite element mesh.

The use of adjoint equations allows the computation of derivatives with respect to large parameter sets in shape optimization problems where the thickness and midsurface of the shell are computed so as to generate a radiated sound field subject to broad-band design requirements. Numerical examples are presented.

## Multiple Traces Boundary Integral Formulation for Helmholtz Transmission Problems

C. Jerez–Hanckes
ETH Zürich, Switzerland

We present a boundary integral formulation of the Helmholtz transmission problem for composite scatterers (piecewise constant coefficients) that lends itself to operator preconditioning via Calderón projectors. The method relies on local traces on subdomains and weak enforcement of transmission conditions. The variational formulation is set in Cartesian products of standard Dirichlet and special Neumann trace spaces, for which restriction and extension by zero operations are well defined. In particular, the Neumann trace spaces over each subdomain boundary are built as piecewise $\widetilde{H}^{-1/2}$-distributions over each associated interface. Through the use of interior Calderón projectors, the problem is cast in variational Galerkin form with a matrix operator whose diagonal is composed of block boundary integral operators. We show existence and uniqueness of solutions based on an extension of Lion's projection lemma for non-closed subspaces. Numerical experiments in 2-D validate the method when compared to alternative approaches and show its amenability to different types of preconditioning.

## $L_2$ error estimates for a BEM–based FEM

C. Hofreither
Johannes Kepler Universität Linz, Austria

We present a non-standard finite element method based on element-local boundary integral operators that permits polyhedral element shapes as well as meshes with hanging nodes. The method employs elementwise PDE-harmonic trial functions and can thus be interpreted as a local Trefftz method. We review results on error estimates in the $H^1$-norm. By passing from a non-conforming primal formulation to a conforming mixed formulation incorporating both Dirichlet and Neumann traces as unknowns, we are able to derive new $L_2$ error estimates.

## Application of Interpolation Theory to Adaptive 3D-BEM

M. Karkulik, D. Praetorius
TU Wien, Austria

Recently, we proved in [1] the convergence of an adaptive boundary element method for the Dirichlet problem in two dimensions. In our talk, we show how the analysis therein can be extended to three dimensions. We first show how the K-Method of the theory of interpolation spaces can be used to obtain approximation properties of a certain class of quasi-interpolation operators in fractional order Sobolev spaces, even for adaptively generated meshes. Then, we use this approach to show convergence of a data-perturbed boundary element method for the Dirichlet problem in three dimensions.

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**Iterative coupling of DEM–BEM regions with an overlapping FEM zone**

G. Karlis, L. Malinowski, G. Beer, J. Rojek
TU Graz, Austria

One of the characteristics of the numerical simulation in geotechnical engineering is that nonlinear/discontinuous behaviour is concentrated on small portions of the total domain. It is not very efficient to use volume based methods for the analysis of the whole domain. On the other hand a discretisation of this domain into distinct elements is also inefficient because the zone of interest, where discontinuous behaviour occurs, is quite small.
The aim of the current work is to develop a simulation methodology that allows the solution of multiregion elastostatic problems using different numerical methods (BEM/FEM/DEM), coupled iteratively.
During this talk an iterative algorithm for coupling two or more BEM subdomains will be outlined and serve as a base for coupling static BEM with a dynamic DEM code. To achieve this a FEM overlapping area has been adopted in order to simplify the whole procedure.

## Differential Forms and Boundary Integral Equations

S. Kurz[1], B. Auchmann[2]
[1]TU Tampere, Finland, [2]CERN, Geneva, Switzerland

In recent years, a remarkable amount of papers has been published that treat continuous and discrete electromagnetics in terms of differential forms. However, most of these papers focus on finite element and finite difference methods. There are only a few contributions that deal with the boundary element method in this setting.
The aim of the present paper is to show how integral equations of the electromagnetic theory can be expressed in the language of differential forms. A generic second order Helmholtz type problem is considered, in terms of differential forms, which is defined on a sub-domain $\Omega$ of Euclidean space, with Lipschitz boundary $\Gamma$. The problem encompasses and generalizes the scalar and vector Helmholtz-type problems. In fact, scalar fields and vector fields, both in the domain and on the boundary, are obtained from differential forms by metric-induced translation isomorphisms. The paper explains in each step how the general results in the differential-form framework relate to their well-known scalar and vector counterparts, by means of the translation isomorphisms.
As analytic framework, the Sobolev spaces $H^{0,p}(d,\Omega)$ play a central role. They contain square integrable differential $p$–forms whose exterior derivative is square integrable, too. The related trace spaces and the Hodge duals of these spaces are presented as well.
Starting from the fundamental solution of the scalar Helmholtz equation, a fundamental solution for the generic problem is constructed, in terms of a double form. Sloppily speaking, double forms are forms in one space with coefficients that are forms in another space. Green's first and second identities can be stated for differential forms. With the fundamental solution, a representation formula can be derived. For $p = 0$ and $p = 1$, respectively, the translation isomorphisms recover the Kirchhoff and Stratton-Chu representation formulas.
Single and double layer potentials can be identified, and their traces yield the Calderón projector, which consists of single layer, double layer and hypersingular boundary integral operators. The properties of these operators are studied in the Sobolev space framework, and a variational formulation is examined.
Since differential forms possess discrete counterparts, the discrete differential forms, such schemes lend themselves naturally to discretisation. Consequently, boundary element techniques can be reinterpreted in terms of discrete differential forms. Some examples are considered, to demonstrate the advantages of this viewpoint.

**Optimal TBETI for Multi–Body Contact Problems**

Z. Dostal, T. Kozubek, M. Sadowska, D. Lukas
VSB TU Ostrava, Czech Republic

We consider a multi-body elastic contact problem with Tresca friction. The linear elastic problem with mixed boundary conditions is formulated as a boundary integral equation in terms of the Steklov-Poincare (SP) operator. We employ Galerkin boundary element discretization and the Total-BETI domain decomposition approach, which leads to a minimization problem with both linear equality and inequality constaints and the objective functional involving a dissipative Tresca term. Applying the duality concept we arrive at a saddle-point problem, where the linear inequality constraints transfer to simple bound constraints and the nonlinear Tresca friction term after a regularization translates to additional separable quadratic constraints. When solving the problem in parallel, each slave-process is responsible for actions of its local SP operator and the pseudoinverse. We accelerate both by the Adaptive Cross Approximation (ACA) technique, while the pseudoinverse (Neumann-to-Dirichlet map) is replaced by iterative CG-solution for regularized Neumann problem. The outer CG-iterations of the dual quadratic problem are preconditioned by the projector to the rigid body modes. Finally, we give theoretical results which guarantee numerical scalability of our algorithm and document performance of the method on numerical results for real-life engineering benchmark problems.

**A fast directional multilevel summation method for oscillatory kernels based on Chebyshev interpolation and adaptive cross approximation**

Ma. Messner[1], E. Darve[2]
[1]TU Graz, Austria, [2]Stanford University, USA

Many applications lead to large systems of linear equations with dense matrices. Direct matrix-vector products become prohibitive, since the computational cost increases quadratically with the size of the problem. By exploiting specific kernel properties fast algorithms can be constructed.

A directional multilevel algorithm for translation-invariant **oscillatory kernels** of the type $K(x,y) = G(x-y)\, \mathrm{e}^{\imath k|x-y|}$, with $G(x-y)$ being any smooth kernel, will be presented. We will first present a general approach to build fast multipole methods (FMM) based on Chebyshev interpolation and the adaptive cross approximation (ACA) for **smooth kernels**. The Chebyshev interpolation is used to transfer information up and down the levels of the FMM. The scheme is further accelerated by compressing the information stored at Chebyshev interpolation points using ACA and QR decompositions. This leads to a nearly optimal computational cost with a small pre-processing time due to the low computational cost of ACA. This approach is in particular faster than performing singular value decompositions.

This does not address the difficulties associated with the oscillatory nature of $K$. For that purpose, we consider the following modification of the kernel $K^u = K(x,y)\, e^{-\imath k u\cdot(x-y)}$, where $u$ is a unit vector. We proved that the kernel $K^u$ can be interpolated efficiently when $x-y$ lies in a cone of direction $u$. This result is used to construct an FMM for the kernel $K$.

Theoretical error bounds will be presented to control the error in the computation as well as the computational cost of the method. The talk ends with the presentation of 2D and 3D numerical convergence studies, and computational cost benchmarks.

**Spectral Galerkin Method for Surface Integral Equations on Nanoparticles in Three Dimensions**[1]

L. Raguin[1], R. Vogelgesang[2], C. Hafner[1]
[1]ETH Zürich, Switzerland
[2]Max–Planck–Institut für Festkörperforschung Stuttgart, Germany

The most adventurous scientists in experimental studies as well as the most experienced theoreticians in academic research have been intrigued and challenged by optics for centuries. In this talk, the development of a new Spectral Galerkin Method (SGM) for fast and accurate three-dimensional (3D) analysis of the optical properties of nanoparticles illuminated by a light source, thereby bringing a new discovery into today's nanooptics, will be discussed.
An important problem arising in today's nanooptics is to solve the Maxwell's equations when all media are characterized by the frequency-dependent complex dielectric permittivity and magnetic permeability [1]. One may reduce this problem to Surface Integral Equations (SIEs) through the use of *layer potentials* [2] when the electric and magnetic fields are expressed in terms of scalar and vector potentials [1]. These SIEs feature singular integral kernels. It may give rise to ill-conditioned discrete operators. We propose to overcome these difficulties by using a *spectral* discretization scheme, the analog of that already has proved its viability for the fast and efficient numerical treatment of numerous 2D problems in nanooptical applications [3]. First, with the surface parameterization we change the variables of integration converting SIEs to the ones over a unit sphere. Second, the Galerkin's method with *spherical harmonics* as the approximating functions is applied to these new equations. Third, the integral kernels are regularized analytically. This step also converts all equations to Fredholm integral equations of the second kind. Finally, the Galerkin's coefficients are calculated by fast transforms for spherical harmonic expansions, leading to fast numerics in addition to high accuracy.
SGM leads to smaller linear system in comparison with classical BEM. The complexity for 3D problems is of the same order as the one of classical FEM for 2D problems. In the past such a global approximation procedures would be less flexible or general than FEM and BEM because of the restriction to surfaces with known parameterization [4]. However, due to recent progress in development of *bijection mappings* for surfaces of nanoparticles on a sphere our SGM is applicable to study all surfaces in nanooptical applications. In addition, contrary to the approach used in [4], we propose to regularize integral operators by subtracting the layer potentials on a unit sphere. Then, new regularization procedure takes into account the complicate dispersive behavior of plasmonic materials at nanoscale making this approach best suited for nanooptical applications.

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[1]This work is supported financially by Swiss National Science Foundation project no. 200021-119976 "Spectral Galerkin Boundary Integral equation methods for plasmonic nanostructures". Helpful advices of Prof. R. Hiptmair and Prof. R. Vahldieck are gratefully acknowledged.

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## Time domain BEM: Brief overview and trends

M. Schanz
TU Graz, Austria

Boundary element formulations in time domain are well established in the engineering and the mathematical literature, for an overview see [2]. In principle three types of formulations can be found:

- Direct in time domain with analytical integration of the time convolution
- Calculation in Laplace or Fourier domain with a subsequent numerical inverse transformation
- Formulations based on the Convolution Quadrature Method (CQM)

The latter formulation goes back to [3,4] and can either be formulated as a true time stepping method or, as proposed by [2], as a calculation of decoupled Laplace domain problems with an inverse transformation. Opposite to the usual formulation in the transformed domain, here, the time step size is the only parameter to be adjusted. Hence, a physical interpretable parameter which follows the CFL-condition is used and not some sophisticated parameters like in inverse Laplace transforms.
In the present talk, the above mentioned procedures are very briefly recalled at either the example of elastodynamics or acoustics. Then the CQM based approached is discussed in more detail and some examples are given. Finally, the way to introduce fast BEM techniques is sketched.

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**Variational inequalities and boundary element methods**

O. Steinbach
TU Graz, Austria

Variational inequalities to be considered in $H^{1/2}(\Gamma)$ result from different applications, e.g., boundary value problems with boundary conditions of Signorini type, contact problems in elasticity, or from constrained optimal control problems. In this talk will will discuss both the error analysis of the boundary element solution, and iterative solution strategies.

**Fast solvers for the hp–version boundary element method and applications in electromagnetics**

E. P. Stephan

Leibniz Universität Hannover, Germany

We present new results from [1,2] on various Schwarz methods for the h and p versions of the boundary element method applied to prototype first kind integral equations on surfaces. When those integral equations (weakly/hypersingular) are solved numerically by the Galerkin boundary element method, the resulting matrices become ill-conditioned. Hence, for an efficient solution procedure appropriate preconditioners are necessary to reduce the numbers of CG-iterations. In the p version where accuracy of the Galerkin solution is achieved by increasing the polynomial degree the use of suitable Schwarz preconditioners (presented in the paper) leads to only polylogarithmically growing condition numbers. For the h version where accuracy is achieved by reducing the mesh size we present a multi-level additive Schwarz method which is competitive with the multigrid method. Applications are given in electromagnetics for solving the eddy current problem or the electrical field integral equation using FEM and BEM.

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## A boundary element method for Laplacian eigenvalue problems

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For the solution of Laplacian eigenvalue problems we propose a boundary element method which is used to solve equivalent nonlinear eigenvalue problems for related boundary integral operators. The discretization of the boundary integral operator eigenvalue problems leads to algebraic nonlinear eigenvalue problems. We use a recently proposed method [1] which reduces the algebraic nonlinear eigenvalue problems to linear ones. The method is based on a contour integral representation of the resolvent operator and it is suitable for the extraction of all eigenvalues in predefined interval which is enclosed by the contour. The dimension of the resulting linear eigenvalue problem corresponds to the number of eigenvalues which lie inside the contour. The main numerical effort consists in the evaluation of the resolvent operator for the contour integral which requires the solution of several linear systems. Compared with other methods for nonlinear eigenvalue problems no initial approximations of the eigenvalues and eigenvectors are needed. First numerical examples demonstrate the robustness of the method.

### References

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## Comparison of boundary element methods for magnetostatic field problems

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We consider transmission problems of the potential equation with piecewise constant coefficients appearing in, e.g., the modelling of electric fields in dielectric media and in scalar potential formulations in magnetostatics. We compare a class of global indirect boundary integral formulations to a a domain decomposition approach to solve these transmission problems by fast boundary element methods, e.g., the Fast Multipole Method.

We discuss the pros and cons of the considered formulations and compare the performance, the accuracy and the stability of the approaches for several numerical examples of industrial applications.

**Stabilized Boundary Element Formulation for Maxwell**

L. Weggler
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The solution of the harmonic Maxwell equations by a conventional variational formulation (FEM or BEM) requires a different treatment when it comes to wave numbers close to zero. In order to solve the so called low frequency problem one has to stabilize the formulation by incorporating Gauß law. A theoretical and numerical study of this subject is presented.

**Adaptive BEM based FEM on general polygonal meshes and residual error estimators**

S. Weißer

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We briefly introduce a special finite element method that solves the stationary isotropic heat equation with Dirichlet boundary conditions on arbitrary polygonal and polyhedral meshes. The method uses a space of locally harmonic ansatz functions to approximate the solution of the boundary value problem. These ansatz functions are constructed by means of boundary integral formulations. Due to this choice, the proposed finite element method can be used on general polygonal non-conform meshes. Hanging nodes are treated quite naturally and the material properties are assumed to be constant on each element.

In a second step we focus on uniform and adaptive mesh refinement. One important point is the treatment of these arbitrary elements. We propose a method to refine polygonal bounded elements which are convex.

In order to do adaptive mesh refinement it is essential to look at a posteriori error estimates. Standard methods are based on triangular or quadrilateral meshes. The challenging part is to handle the arbitrary polygonal and polyhedral meshes. We generalize the ideas of residual error estimators and use them in numerical examples.

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**BETI methods for scattering problems**

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In this talk we want to present basic ideas for a tearing and interconnecting approach for acoustic and electromagnetic scattering, using boundary integral equations on the local subdomains. The tearing and interconnecting approach is normally used for partial differential equations which lead to elliptic bilinear forms. Nevertheless, C. Farhat introduced the FETI also for the Helmholtz equation (using FEM instead of BEM on the local subdomains), now called FETI-H. In this talk we describe, how this approach can be used for acoustic and electromagnetic scattering problems in combination with the boundary element method. Instead of standard transmission boundary conditions of Dirichlet and Neumann type we may use Robin type interface conditions, which result in a stable formulation which is robust to possible spurious modes.

**Application of Hierarchical Matrices to a BEM Plasticity Algorithm in Tunneling**

J. Zechner, G. Beer
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In the design process of tunnels the geotechnical engineer usually selects a suitable alignment, decides how and in which sequence the excavation process is done and which support measures such as rockbolts and shotcrete should be implemented. For this purpose we propose the use of the Boundary Element Method (BEM).
A Collocation-BEM formulation in elasto-statics is used to simulate the surrounding rock-mass of the tunnel and the tunnel support. Therefor, small strain plasticity is considered and the evaluation of stresses in some regions of the domain is necessary. The numerical complexity in terms of computation and storage for the additional boundary integral equation is of $\mathcal{O}(n^2)$ where $n$ denotes the number of stress computations. In terms of numerical efficiency compared to the Finite Element Method the implementation is not competitive unless so-called "Fast Methods" are applied. Hierarchical Matrices and Adaptive Cross Approximation are applied to the chosen formulation. In this presentation intermediate results of the ongoing implementation are shown.

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