Space–Time Methods

6. For given $u \in H^{1/2}_{0,}(0,T)$, find $w \in H^{1/2}_{,0}(0,T)$ as solution of the variational problem

$$\langle w, z \rangle_{H^{1/2}_{,0}(0,T)} = \langle \partial_t u, z \rangle_{(0,T)} \quad \text{for all } z \in H^{1/2}_{,0}(0,T).$$

7. Let $u \in H^1_{0,}(0,T)$ be given. Determine the solution $w \in H^1_{,0}(0,T)$ of the variational problem

$$\int_0^T \partial_t w(t) \partial_t v(t)\, dt = -\int_0^T \partial_t u(t) \partial_t v(t)\, dt \quad \text{for all } v \in H^1_{,0}(0,T)$$

by using the ansatz

$$w(t) = \sum_{k=0}^{\infty} w_k \cos\left(\left(\frac{\pi}{2} + k\pi\right)\frac{t}{T}\right).$$

8. Find a closed representation of the solution w as computed in **12.** in terms of u.

9. Compute the solution of the ordinary differential equation ($\mu > 0$)

$$\partial_{tt} u(t) + \mu u(t) = f(t) \quad \text{for } t > 0, \quad u(0) = \partial_t u(t)_{|t=0} = 0$$

by rewriting the differential equation as first order system.