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SS 2022
Blatt 5
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Space–Time Methods

19. Compute the solution of the wave equation

$$\partial_{tt}u(x,t) - \partial_{xx}u(x,t) = 0 \quad \text{for } x \in \mathbb{R},\ t > 0,$$

with the initial conditions

$$u(x,0) = \varphi(x), \quad \partial_t u(x,t)_{|t=0} = \psi(x) \quad \text{for } x \in \mathbb{R}.$$

20. Use a suitable series representation to compute the solution of the wave equation

$$\partial_{tt}u(x,t) - \partial_{xx}u(x,t) = 0 \quad \text{for } x \in (0,1),\ t > 0$$

with the initial conditions

$$u(x,0) = \varphi(x), \quad \partial_t u(x,t)_{|t=0} = 0 \quad \text{for } x \in (0,1).$$

21. Consider the solution u as determined in **20.** What are the assumptions on φ to ensure $u \in H^1(Q)$, where $Q := (0,1) \times (0,T)$.

22. Compute the solution of the ordinary differential equation ($\mu > 0$)

$$\partial_{tt}u(t) + \mu u(t) = f(t) \quad \text{for } t > 0, \quad u(0) = \partial_t u(t)_{|t=0} = 0$$

by rewriting the differential equation as first order system.

23. Compute the solution of the transmission problem

$$\partial_{tt}u_-(x,t) - \partial_{xx}u_-(x,t) = 0 \quad \text{for } x < 0,\ t > 0,$$

$$\partial_{tt}u_+(x,t) - \partial_{xx}u_+(x,t) = 0 \quad \text{for } x > 0,\ t > 0,$$

with the transmission conditions

$$u_+(0,t) - u_-(0,t) = g(t), \quad \partial_x u_+(x,t)_{|x=0} - \partial_x u_-(x,t)_{|x=0} = 0 \quad \text{for } t > 0,$$

and with the initial conditions

$$u_\pm(x,t) = \partial_t u_\pm(x,t)_{|t=0} = 0, \quad x \in \mathbb{R}_\pm.$$

Here we assume that g satisfies the compatibility condition $g(0) = 0$.