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Technische Numerik 2

Exercise sheet 1, March 11, 2020

Exercise 1:

a) Sketch the function $u_h = u_{0,h} + \tilde{g}$ with $\tilde{g}(x) = 1 - x$,

$$u_{0,h}(x) = \sum_{k=1}^{2} u_{0,k}\varphi_k^1(x),$$

the coefficients $u_{0,1} = u_{0,2} = 1/3$ and the grid points $x_k = k/3$, $k = 0, \dots, 3$. φ_k^1 denote the basis functions of $S_h^1(0,1)$.

b) Evaluate the function $u_h(x)$ in $x = 1/4$.

Exercise 2: In the interval $[y_0, y_2]$ the Lagrangian basis of quadratic polynomials is given by

$$L_0^2(x) = \frac{x-y_1}{y_0-y_1}\frac{x-y_2}{y_0-y_2}, \quad L_1^2(x) = \frac{x-y_0}{y_1-y_0}\frac{x-y_2}{y_1-y_2}, \quad L_2^2(x) = \frac{x-y_0}{y_2-y_0}\frac{x-y_1}{y_2-y_1}.$$

There holds $L_k(x_j) = \delta_{kj}$.

Grid points are given by $x_j = j/4$ for $j = 0, \dots, 4$. For the two intervals $\tau_1 = [x_0, x_2]$ and $\tau_2 = [x_2, x_4]$ we define the finite elment space $S_h^2(0,1)$ of piecewise quadratic and globally continiuous functions with basis:

$$\varphi_0^2(x) = \begin{cases} L_0^2(x) & \text{for } x \in \tau_1 \\ 0 & \text{for } x \in \tau_2 \end{cases} \qquad \varphi_1^2(x) = \begin{cases} L_1^2(x) & \text{for } x \in \tau_1 \\ 0 & \text{for } x \in \tau_2 \end{cases} \qquad \varphi_2^2(x) = \begin{cases} L_2^2(x) & \text{for } x \in \tau_1 \\ L_0^2(x) & \text{for } x \in \tau_2 \end{cases}$$

$$\varphi_3^2(x) = \begin{cases} 0 & \text{für } x \in \tau_1 \\ L_1^2(x) & \text{für } x \in \tau_2 \end{cases} \qquad \varphi_4^2(x) = \begin{cases} 0 & \text{for } x \in \tau_1 \\ L_2^2(x) & \text{for } x \in \tau_2 \end{cases}$$

Here we set for τ_1: $y_0 = x_0$, $y_1 = x_1$, $y_2 = x_2$. While we set for τ_2: $y_0 = x_2$, $y_1 = x_3$, $y_2 = x_4$.

a) Sketch the function

$$v_h(x) = \sum_{k=0}^{4} v_k\varphi_k^2(x),$$

with coefficients $v_0 = 0$, $v_1 = 1/2$, $v_2 = 2$, $v_3 = 3/2$ and $v_4 = 1/2$.

b) Evaluate the function $v_h(x)$ in $x = 1/4$ and $x = 1/8$.

Exercise 3: Simplify the algorithm of the LU decomposition without pivoting for the triangular matrix

$$\begin{pmatrix} a_1 & b_1 & & & \\ c_2 & a_2 & b_2 & & \\ & \ddots & \ddots & \ddots & \\ & & c_{n-1} & a_{n-1} & b_{n-1} \\ & & & c_n & b_n \end{pmatrix} = \begin{pmatrix} 1 & & & & \\ l_{2,1} & 1 & & & \\ & \ddots & \ddots & & \\ & & l_{n-1,n-2} & 1 & \\ & & & l_{n,n-1} & 1 \end{pmatrix} \begin{pmatrix} r_{1,1} & b_1 & & & \\ & r_{2,2} & b_2 & & \\ & & \ddots & \ddots & \\ & & & r_{n-1,n-1} & b_{n-1} \\ & & & & r_{n,n} \end{pmatrix}.$$

Please assume that none of the diagonal entries becomes zero during the algorithm. What is the asymptotic complexity of the simplified LU decomposition?