Technische Universität Graz
Institut für Angewandte Mathematik
Assoc. Prof. Dr. Günther Of

Numerics and Simulation
Elective subject mathematics (DDM)

Exercise sheet 2, March 23, 2022

Exercise 6: Consider the boundary value problem

$$-u''(x) = 0 \quad \text{in } \Omega = (0,1), \quad u(0) = u(1) = 0.$$

Find the condition on $\theta \in [0,1]$ and $a \in (0,1)$, such that the Dirichlet Neumann method converges for the initial guess $u^0(a) = 1$ and the decomposition into two subdomains $\Omega_1 = (0,a)$ and $\Omega_2 = (a,1)$. Hint: Solve the local problems explicitely.

Exercise 7: Formulate the variational problem for the mixed boundary value problem

$$-\Delta u = f \quad \text{in } \Omega, \quad u = 0 \quad \text{on } \Gamma_D, \quad \frac{\partial u}{\partial n} = g \quad \text{on } \Gamma_N$$

where $\partial\Omega = \overline{\Gamma}_D \cup \overline{\Gamma}_N$, $\Gamma_D \cap \Gamma_N = \emptyset$ and $\operatorname{meas}\Gamma_D \neq 0$. Formulate the Dirichlet Neumann method for two subdomains in terms of the related local boundary value problems. Find a condition such that the local boundary value problems are uniquely solvable.

Exercise 8: State suitable variational formulations and the related systems of linear equations (for piecewise linear and globally continuous basis functions) for the local boundary value problems of Exercise 7.

Exercise 9: Interpret the Neumann Dirichlet method for the Dirichlet boundary value problem as Richardson iteration for the flux formulation (1.17). Use the assumption $\partial\Omega_i \cap \partial\Omega \neq \emptyset$ for $i = 1, 2$.

Exercise 10: Find the condition for the convergence of the Neumann Dirichlet method, if $\partial\Omega_2 \cap \partial\Omega \neq \emptyset$ and

$$(S^{(1)}\underline{w}, \underline{w}) \leq c_2(S^{(2)}\underline{w}, \underline{w}) \qquad \forall \underline{w} \in \mathbb{R}^N.$$