

Numerics and Simulation

Elective subject mathematics (DDM)

Exercise sheet 7, June 8, 2022

Exercise 29: Consider the Dirichlet boundary value problem

$$-u''(x) = f(x) \quad \text{for } x \in (0, 1); \quad u(0) = u(1) = 0.$$

This problem is solved in `simplefem.c` by a finite element method. Implement a direct solver for symmetric tridiagonal matrices (see Ex. 5). Describe your implementation briefly. Proof its correctness by providing the resulting vector for $N = 8$.

Exercise 30: Parallelize `simplefem.c` using MPI by distributing the elements connected but non-overlapping to the processes and by implementing the related parallel version of the CG method for distributed matrices. Show the correctness of the parallel version for $N = 16$ and $p = 4$ subdomains by providing the resulting vector .

Determine the numbers of iterations of the parallel CG method for $N = 512$ and $N = 1024$ and several numbers p of subdomains:

N, p	2	4	8
512			
1024			

Which conclusions can be drawn from these numbers?

Exercise 31: Implement a parallel version of the additive 1-level overlapping Schwarz preconditioner (i.e. without coarse grid). Use the direct solver of Ex. 29 as local exact solver. It is sufficient to implement an overlap up to half of the elements of a subdomain, such that the communication with the neighbors is sufficient.

Determine the numbers of iterations of the parallel preconditioned CG method for the following combinations of the number p of subdomains and an overlap of η elements for $N = 512$

η, p	2	4	8	16	32	64
1						
2						
4						
8						×
16					×	×
32				×	×	×

and $N = 1024$:

η, p	2	4	8	16	32	64
1						
2						

Which conclusions can be drawn from these numbers?

Exercise 32: Enhance your additive Schwarz preconditioner (Ex. 31) by a coarse grid solver to an additive 2-level Schwarz preconditioner. For simplicity, use the boundary nodes of the subdomains as degrees of freedom for the coarse grid problem and a parallel unpreconditioned CG method as coarse grid solver. Note that convergence of the inner CG method may fail in the last steps of the outer CG iteration.

Determine the numbers of iterations of the parallel preconditioned CG method for $N = 512$ and the following combinations of the number p of subdomains and the overlap of η elements:

η, p	2	4	8	16	32	64
1						
2						
4						
8						×
16					×	×
32				×	×	×

Which conclusions can be drawn from these numbers?