Technische Universität Graz
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Numerical Mathematics 4

Exercise sheet 5, December 12, 2024

Exercise 19: Determine the three shape functions of the lowest order Raviart Thomas elements on the reference element $(0,0),(1,0),(0,1)$. Sketch the related vector fields.

Sketch the vector field of the Raviart Thomas basis function of the common edge of the two triangles with corner points $(0,0),(1,0),(0,1)$ and $(1,0),(1,1),(0,1)$, respectively. Check the normal continuity of this basis function along the common edge.

Exercise 20: Prove that for $\hat{\vec{v}} = |\det B_T| B_T^{-1} \vec{v} \circ F_T$ and any $T \in \mathcal{T}_h$ holds

i) $\hat{\vec{v}} \in [H^1(\hat{T})]^d \Rightarrow \vec{v} \in [H^1(T)]^d$,

ii) $\vec{v} \in RT(T) \Leftrightarrow \hat{\vec{v}} \in RT(\hat{T})$,

where $\hat{}$ denotes the quantities on the reference element $\hat{T}$.

Exercise 21: Let $\vec{v} \in [H^1(T)]^d$, $q \in H^1(T)$. For $\hat{\vec{v}} = |\det B_T| B_T^{-1} \vec{v} \circ F_T$ and $\hat{q} = q \circ F_T$ there holds

i) $\int_{\hat{T}} \hat{\vec{v}} \cdot \nabla \hat{q} \, d\hat{x} = \int_T \vec{v} \cdot \nabla q \, dx$

ii) $\hat{\operatorname{div}} \, \hat{\vec{v}} = |\det B_T| \operatorname{div} \vec{v}$

iii) $\int_{\hat{T}} \hat{q} \, \hat{\operatorname{div}} \, \hat{\vec{v}} \, d\hat{x} = \int_T q \operatorname{div} \vec{v} \, dx$

iv) $\int_{\partial \hat{T}} \hat{q} \, \hat{\vec{v}} \cdot \hat{\vec{n}}_{\hat{T}} \, ds_{\hat{x}} = \int_{\partial T} q \, \vec{v} \cdot \vec{n}_T \, ds_x$

Exercise 22: Proof the unique solvability of the dual mixed formultation of Exercise 18.