Technische Universität Graz

9th Austrian Numerical Analysis Day

Graz, April 11–12, 2013

G. Of, O. Steinbach (eds.)

Berichte aus dem Institut für Numerische Mathematik

Book of Abstracts 2013/2

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Technische Universität Graz
Institut für Numerische Mathematik
Steyrergasse 30
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Time Schedule

Thursday, April 11, 2013

12:30		Registration
12:50		Opening
13:00	Andreas Schröder	*hp*-adaptive FEM with biorthogonal basis functions for elliptic obstacle problems
13:20	Michael Feischl	Adaptive FEM with optimal convergence rates for a certain class of non-symmetric problems
13:40	Lukas Einkemmer	A discontinuous Galerkin discretization for the Vlasov-Poisson equations
14:00	Martin Neumüller	Analysis of a two grid space-time solver
14:20	Monika Wolfmayr	Robust Multilevel Preconditioning for Heterogeneous Reaction-Diffusion Problems
14:40		Coffee Break
15:10	Thomas Glatz	The Inverse Source Problem for the Helmholtz Equation and Photoacoustic Tomography
15:30	Hermann Mena	On the Numerical Solution of Large-Scale Riccati Equations
15:50	Gerhard Unger	Boundary element methods for acoustic resonance problems
16:10	Othmar Koch	An Approximate Eigensolver for Self-Consistent Field Calculations
16:30		Coffee Break
17:00	Thomas Führer	Stability of FEM-BEM couplings for nonlinear elasticity problems
17:20	Manfred Liebmann	Solving the Time-Dependent Schrödinger Equation on CPU and GPU Clusters
17:40	Harald Hofstätter	A Perl program for the symbolic manipulation of flows of differential equations and its application to the analysis of defect-based error estimators for splitting methods
19:00		Dinner

Friday, April 12, 2013

8:30	Nadir Bayramov	Finite element methods for transient convection-diffusion equations with small diffusion
8:50	Markus Faustmann	$\mathcal{H}$-matrix approximability of inverse FEM matrices for various boundary conditions
9:10	Christoph Augustin	Strongly scalable numerical methods to simulate cardio-vascluar tissues
9:30	Elias Karabelas	Hybrid cG/dG Finite Element Methods for Solving Semilinear Parabolic Equations
9:50	Antti Hannukainen	Two-scale homogenization of the eddy-current problem in laminated media
10:10		Coffee Break
10:40	Wolfgang Krendl	An Efficient Robust Solver for Optimal Control Problems for the Stokes Equations in the Time-Harmonic Case
11:00	Gerhard Kitzler	A High Order Discontinuous Galerkin Method for the Boltzmann equation
11:20	Petra Csomós	Exponential integrators for shallow water equations
11:40	Erika Hausenblas	A perturbation result for quasi-linear stochastic differential equations in UMD Banach spaces
12:00	Monika Dörfler	Numerical aspects of audio processing
12:20		Lunch Break
14:00	Hans G. Feichtinger	Numerical Questions in Time-frequency Analysis
14:20	Winfried Auzinger	Local error structures of higher-order exponential splitting schemes
14:40	Marcus Page	A general integrator for the Landau-Lifshitz-Gilbert equation
15:00	Arno Kimeswenger	Boundary control of exterior boundary value problems
15:20	Boris Vexler	New regularity results and improved error estimates for optimal control problems with state constraints
15:40		Closing

Abstracts in Chronological Order

hp-adaptive FEM with biorthogonal basis functions for elliptic obstacle problems

Andreas Schröder[1], Lothar Banz[2]

[1] University of Salzburg
[2] Leibniz University Hanover

The talk presents an hp-adaptive mixed finite element discretiziation for a non-symmetric elliptic obstacle problem where the dual space is discretized via biorthogonal basis functions. The resulting algebraic system only includes box constraints and componentwise complementarity conditions. This special structure is exploited to apply efficient semi-smooth Newton methods using a penalized Fischer-Burmeister NCP-function in each component. Adaptivity is accomplished via a posteriori error control which is also introduced. Several numerical experiments confirm the applicability of the hp-adaptive scheme.

Adaptive FEM with optimal convergence rates for a certain class of non-symmetric problems

Michael Feischl, Thomas Führer, Dirk Praetorius

Vienna University of Technology

In this talk based on the recent preprint [2], we analyze adaptive mesh-refining algorithms for conforming finite element discretizations of second-order partial differential equations, i.e.

$$\begin{aligned} \mathcal{L}u(x) := -\mathrm{div}\boldsymbol{A}(x)\nabla u(x) + \boldsymbol{b}(x)\cdot\nabla u(x) + c(x)u(x) &= f(x) \quad && x \in \Omega, \\ u(x) &= 0 && x \in \partial\Omega \end{aligned}$$

posed on a Lipschitz domain Ω. For a given mesh $\mathcal{T}_\ell$, we allow continuous $\mathcal{T}_\ell$-piecewise polynomials of arbitrary, but fixed polynomial order with homogeneous boundary conditions $\mathcal{S}_0^p(\mathcal{T}_\ell)$ as ansatz functions. The adaptivity is driven by the standard residual error estimator ρ_ℓ. We prove convergence even with quasi-optimal algebraic convergence rates. This means that if, given an optimal sequence of meshes $\widetilde{\mathcal{T}}_\ell$ with corresponding error estimators $\widetilde{\rho}_\ell$, a convergence rate of $s > 0$ is possible, i.e.

$$\widetilde{\rho}_\ell \le \widetilde{C}(\#\widetilde{\mathcal{T}}_\ell - \#\widetilde{\mathcal{T}}_0)^{-s} \quad \text{for all } \ell \in \mathbb{N},$$

then the adaptive Algorithm generates meshes $\mathcal{T}_\ell$ with corresponding error estimators ρ_ℓ and Galerkin approximations U_ℓ, which reveal the same rate of convergence, i.e.

$$C_{\mathrm{rel}}^{-1}\|\nabla(u - U_\ell)\|_{L^2(\Omega)} \le \rho_\ell \le C(\#\mathcal{T}_\ell - \#\mathcal{T}_0)^{-s} \quad \text{for all } \ell \in \mathbb{N}.$$

The advantages over the state of the art [1] read as follows: Unlike prior works for linear non-symmetric operators, our analysis avoids the artificial quasi-symmetry assumption $\mathrm{div}\boldsymbol{b} = 0$ as well as the interior node property for the refinement. Moreover, the differential operator $\mathcal{L}$ has to satisfy a Gårding inequality only. If $\mathcal{L}$ is uniformly elliptic, no additional assumption on the initial mesh is posed. Finally, our analysis also covers certain nonlinear problems and proves quasi-optimal convergence rates.

References

[1] J. M. Cascón, R. H. Nochetto: Quasi-optimal cardinality of AFEM driven by non-residual estimators, IMA J. Numer. Anal. **32** (2012), 1–29.

[2] M. Feischl, T. Führer, D. Praetorius: Adaptive FEM with optimal convergence rates for a certain class of non-symmetric and possibly non-linear problems, submitted to SIAM J. Numer. Anal. (2012).

A discontinuous Galerkin discretization for the Vlasov–Poisson equations

Lukas Einkemmer[1], Alexander Ostermann[1]

[1] University of Innsbruck

A rigorous convergence analysis of the Strang splitting algorithm with a discontinuous Galerkin approximation in space for the Vlasov–Poisson equations is provided. It is shown that under suitable assumptions the error is of order $\mathcal{O}\left(\tau^2 + h^q + h^q/\tau\right)$, where τ is the size of a time step, h is the cell size, and q the order of the discontinuous Galerkin approximation. In order to investigate the recurrence phenomena for approximations of higher order in space as well as to confirm the stability properties of the method investigated a number of numerical simulations will be shown (including Landau damping and the Molenkamp–Crowley test).

Analysis of a two grid space-time solver

Martin Neumüller[1], Martin J. Gander[2], Olaf Steinbach[1]

[1] Graz University of Technology
[2] Université de Genéve

For evolution equations we present a space-time method based on Discontinuous Galerkin finite elements. Space-time methods have advantages when we have to deal with moving domains and if we need to do local refinement in the space-time domain. For this method we present a multigrid approach based on space-time slaps. This method allows the use of parallel solution algorithms. In particular it is possible to solve parallel in time and space. Numerical examples will be given which show the performance of this space–time multigrid approach.

Robust Multilevel Preconditioning for Heterogeneous Reaction-Diffusion Problems

Monika Wolfmayr[1], Johannes Kraus[2]

[1] Doctoral Program "Computational Mathematics", Johannes Kepler University Linz
[2] Johann Radon Institute for Computational and Applied Mathematics, Johannes Kepler University Linz

This work [2] is devoted to the analysis for constructing robust algebraic multilevel preconditioners for heterogeneous reaction-diffusion problems. We discretize these problems by the finite element method leading to a weighted sum of stiffness and mass matrices. The weighting parameters are often only constant on the subdomains corresponding to the elements of the coarsest mesh partitioning. In order to solve such problems we consider the algebraic multilevel iteration (AMLI) method. The main contribution of this work [2] is to give a rigorous proof that the AMLI method yields a robust and fast solver of optimal complexity for this class of problems. Moreover, we present a time-periodic parabolic optimal control problem as motivation and as a practical example for the relevance of constructing robust AMLI preconditioners for system matrices which are a weighted sum of stiffness and mass matrices. The multiharmonic finite element analysis of such time-periodic parabolic optimal control problems can be found in [3] including an existence and uniqueness proof as well as an estimate for the discretization error. In order to solve the optimal control problem, we state its optimality system and discretize it by the multiharmonic finite element method leading to a system of linear algebraic equations which decouples into smaller systems. In [1], we construct preconditioners for these systems which yield robust and fast convergence rates for the preconditioned minimal residual method. All systems can be solved totally in parallel. We present the results for practically implementing the preconditioners by the AMLI method in [2], which finally leads to a robust and fast solver of optimal complexity.

The research has been supported by the Doctoral Program "Computational Mathematics: Numerical Analysis and Symbolic Computation" under the grant W1214, project DK4, the Johannes Kepler University of Linz and the Federal State of Upper Austria.

References

[1] M. Kollmann, M. Kolmbauer, U. Langer, M. Wolfmayr, W. Zulehner: *A Finite Element Solver for a Multiharmonic Parabolic Optimal Control Problem*. Comput. Math. Appl. 65(3), 2013, 469-486.

[2] J. Kraus, M. Wolfmayr: *Robust Multilevel Preconditioning for Heterogeneous Reaction-Diffusion Problems*. Johannes Kepler University Linz, 2013, *in preparation.*

[3] U. Langer, M. Wolfmayr: *Multiharmonic Finite Element Analysis of a Time-Periodic Parabolic Optimal Control Problem*. Johannes Kepler University Linz, Doctoral Program "Computational Mathematics", 2013, DK-Report No. 2013-01.

The Inverse Source Problem for the Helmholtz Equation and Photoacoustic Tomography

Thomas Glatz[1], Otmar Scherzer[1], Roland Griesmaier[2], Martin Hanke[3]

[1] University of Vienna
[2] University of Leipzig
[3] University of Mainz

This talk discusses the use of photoacoustic imaging methods for solving inverse problems for the Helmholtz equation. The fundamental difference between these two problems is that in the prior the whole frequency spectrum of the wave is available, while in the latter one has to work using only single frequency information. The relation between these two fields is established by narrow-band photoacoustics, where one takes into account limited frequency response of the transducers. Here we reanalyze the effect of a narrow-band recording detector analytically (see also [2, 3]). Additionally, we treat the limit case, assuming to measure just at a single frequency. We present a frequency extension of this data set that leads to a mollified reconstruction, containing valuable information about the support of the source.
On the other hand, the frequency restriction in Photoacoustics corresponds to the near field inverse source problem for the Helmholtz equation, in the sense that we don't make use of the (very common) far field approximation. Moreover, we investigate the connection to [1], were similar Fourier data extensions are used for solving the far field problem.

References

[1] R. Griesmaier, T. Raasch, M. Hanke, Inverse source problems for the Helmholtz equation and the windowed Fourier transform. SIAM J. Sci. Comput., 34 (2012), A1544–A1562.

[2] M. Haltmeier, A mollification approach for inverting the spherical mean Radon transform. SIAM J. Appl. Math., 71 (2011), 1637–1652.

[3] M. Haltmeier and G. Zangerl, Spatial resolution in photoacoustic tomography: effects of detector size and detector bandwidth. Inverse Problems, 26 (2010), 125002.

On the Numerical Solution of Large-Scale Riccati Equations

Hermann Mena[1]

[1] Universität Innsbruck

The numerical treatment of linear quadratic regulator/gaussian design problems for parabolic partial differential equations requires solving large-scale Riccati equations. Typically the coefficient matrices of the resulting equations have a given structure, [1](e.g. sparse, symmetric, low rank ...). We develop efficient numerical methods capable of exploiting this structure and discuss their implementation. The methods are based on a low-rank approximation of the solution and a matrix-valued implementation of the usual ODE methods [2].

References

[1] P. Benner, J.R. Li, T. Penzl: Numerical Solution of Large Lyapunov equations, Riccati Equations, and Linear-Quadratic Control Problems. Numerical Linear Algebra with Applications, Vol. 15, No. 9 (2008), pp. 755-777.

[2] P. Benner, H. Mena: Numerical solution of the Infinite-Dimensional LQR-Problem and the associated Differential Riccati Equations. MPI Magdeburg Preprint, MPIMD/12-13 (2012),

Krylov Subspace Recycling for Families of Shifted Linear Systems

Kirk M. Soodhalter[1], Daniel B. Szyld[2] and Fei Xue [3]

[1] Johannes Kepler University, Linz
[2] Temple University, Philadelphia, [3] University of Louisiana, Lafayette

We address the solution of a sequence of families of linear systems. For the ith family, there is a base coefficient matrix A_i, and the coefficient matrices for all systems in the ith family differ from A_i by a multiple of the identity, i.e.,

$$A_i x_i = b_i \quad \text{and} \quad (A_i + \sigma_i^{(\ell)} I) x_i^{(\ell)} = b_i \quad \text{for} \quad \ell = 1 \ldots L_i,$$

where L_i is the number of shifts at step i. This is an important problem arising in various applications. We extend the method of subspace recycling to solve this problem by introducing a GMRES with subspace recycling scheme for families of shifted systems. This new method solves the base system using GMRES with subspace recycling while constructing approximate corrections to the solutions of the shifted systems at each cycle. These corrections improve the solutions of the shifted system at little additional cost. At convergence of the base system solution, GMRES with subspace recycling is applied to further improve the solutions of the shifted systems to tolerance. We present analysis of this method and numerical results involving systems arising in lattice quantum chromodynamics.

Boundary element methods for acoustic resonance problems

Gerhard Unger[1]

[1] TU Graz

We characterize acoustic resonances as eigenvalues of boundary integral operator eigenvalue problems and apply boundary element methods for their numerical approximation. Eigenvalue problem formulations for resonance problems which are based on standard boundary integral equations exhibit additional eigenvalues which are not resonances but eigenvalues of a related "interior" eigenvalue problem [1]. In practical computations it is for some typical applications hard to extract the resonances when using standard boundary integral formulations. In this talk we present regularized combined boundary integral formulations which only exhibit resonances as eigenvalues. We provide a numerical analysis of the boundary element approximations of these eigenvalue problem formulations and give numerical examples.

References

[1] O. Steinbach, G. Unger: Convergence analysis of a Galerkin boundary element method for the Dirichlet Laplacian eigenvalue problem. SIAM J. Numer. Anal., 50 (2012), 710-728.

An Approximate Eigensolver for Self-Consistent Field Calculations

Othmar Koch[1], Harald Hofstätter[2]

[1] Vienna University of Technology
[2] Vienna University of Technology

We discuss an approximate solution method for the generalized eigenvalue problems arising for instance in the context of electronic structure computations based on density functional theory. The solution method is demonstrated to excel as compared to established solvers in both computational effort and scaling for parallelization. Furthermore, we estimate the error resulting from our proposed subspace method starting from the initial approximations for instance provided in the course of the self-consistent field iteration, showing that in general the approximation quality is improved by our method to yield sufficiently accurate eigenvalues.

Stability of FEM-BEM couplings for nonlinear elasticity problems

Thomas Führer, Michael Feischl, Michael Karkulik, Dirk Praetorius

Vienna University of Technology

We consider a transmission problem in elasticity with a nonlinear material behavior in the bounded interior domain, which can be rewritten by means of the symmetric coupling as well as non-symmetric coupling methods, such as the Johnson-Nédélec coupling. Problems arise when trying to prove solvability of the Galerkin discretization, because the space of rigid body motions is contained in the kernel of the Lamé operator.

In this talk, which is based on the recent preprint [3], we present how to extend the ideas of *implicit stabilization*, developed for Laplace-type transmission problems in [1], to elasticity problems. We introduce modified equations which are fully equivalent (at the continuous as well as at the discrete level) to the original formulations. Solvability of the discrete modified problems, however, hinges on a condition on the discretization space, which states that the space is rich enough to tackle the rigid body motions. We prove that this condition is satisfied for regular triangulations, if the boundary element space contains the piecewise constants.

Our analysis extends the works [2, 4, 5, 6]. Unlike [2], we avoid any assumption on the mesh-size. Unlike [4], we avoid the use of an interior Dirichlet boundary. Unlike [6], we avoid any pre- and postprocessing steps as well as the numerical solution of additional boundary value problems.

References

[1] M. Aurada, M. Feischl, T. Führer, M. Karkulik, J. Melenk, D. Praetorius: Classical FEM-BEM coupling methods: nonlinearities, well-posedness, and adaptivity. Comput. Mech., (2012).

[2] C. Carstensen, S. A. Funken, E. P. Stephan: On the adaptive coupling of FEM and BEM in 2d-elasticity. Numer. Math., 77 (2012), 187–221.

[3] M. Feischl, T. Führer, M. Karkulik, D. Praetorius: Stability of symmetric and nonsymmetric FEM-BEM couplings in nonlinear elasticty. ASC Report 52, TU Wien (2012).

[4] G. Gatica, G. C. Hsiao, F. J. Sayas: Relaxing the hypotheses of Bielak-MacCamy's BEM-FEM coupling. Numer. Math., 120 (2012), 465–487.

[5] G. Of, O. Steinbach: Is the one-equation coupling of finite and boundary element methods always stable? Berichte aus dem Institut für Numerische Mathematik, TU Graz, 6 (2011).

[6] O. Steinbach: On the stability of the non-symmetric bem/fem coupling in linear elasticity. Comput. Mech., (2012).

Solving the Time-Dependent Schrödinger Equation on CPU and GPU Clusters

Manfred Liebmann[1]

[1] Karl–Franzens–Universität Graz

We investigate efficient approximation schemes for solving the Schrödinger equation for time-dependent Hamiltonians on CPU and GPU clusters with applications in optimal control theory.

A Perl program for the symbolic manipulation of flows of differential equations and its application to the analysis of defect-based error estimators for splitting methods

Harald Hofstätter[1]

[1] Vienna University of Technology

For the proof of the asymptotical correctness of certain defect-based a posteriori local error estimators for splitting methods, suitable (integral) representations of local error expansions have to be derived, from which the order of the local error can directly be inferred. In the linear case this is achieved by tedious but relatively straightforward manipulations of operator exponentials. In the nonlinear case, however, to obtain these local error representaions explicitly, complicated expressions involving nonlinear flows of differential equations and higher-order Frechet derivatives of such flows have to be handled. Performing these tedious and error prone calculations manually very quickly becomes unreasonable or even impossible. We describe an especially tailored tool for performing such manipulations symbolically, whose development was strongly facilitated by employing the object-orientedness and the superior string and hash manipulation features of the Perl programming language.

Finite element methods for transient convection-diffusion equations with small diffusion

Nadir Bayramov[1], Johannes Kraus[1]

[1] RICAM, Linz

Transient convection-diffusion or convection-diffusion-reaction equations, with in general small or anisotropic diffusion, are considered. A specific exponential fitting scheme, resulting from finite element approximation, is applied to obtain a stable monotone method for these equations. Error estimates are dicussed for this method and a comparison to the more commonly known SUPG method is drawn. Numerical results are presented (for both methods) including the case of highly anisotropic diffusion tensor.

References

[1] J. Xu, L. Zikatanov, *A monotone finite element scheme for convection-diffusion equations*, Mathematics of Computation, Volume 68, 228 (1999), pp. 1429–1446.

[2] N. Bayramov, J. Kraus, *On the robust solution of convection-diffusion equation coupled with Stokes equation* (in preparation)

$\mathcal{H}$-matrix approximability of inverse FEM matrices for various boundary conditions

Markus Faustmann, Jens Markus Melenk, Dirk Praetorius

Institute for Analysis and Scientific Computing, TU Vienna

The inverse of the finite element (FEM) stiffness matrix corresponding to the Dirichlet problem for elliptic operators with bounded coefficients can be approximated in the data-sparse format of $\mathcal{H}$-matrices with an error that decays exponentially in the block rank employed. This was observed numerically in the PhD thesis of Grasedyck and mathematically shown by Bebendorf & Hackbusch for Dirichlet problems. Such a result is of practical interest, since it provides the mathematical basis to apply $\mathcal{H}$-matrix techniques for black-box preconditioning in iterative solvers.

Possibly due to the method of proof, the existing mathematical analysis has focused on Dirichlet problems. In [1, 2], a new avenue is pursued that permits more general settings. For a BEM setting, for example, [2] shows that the inverses of first kind BEM matrices can be approximated at an exponential rate in the (local) block rank. In this talk, we illustrate that the Dirichlet boundary conditions are not essential and present a corresponding exponential approximability result for the inverses of FEM stiffness matrices arising in problems with Neumann or mixed boundary conditions.

A main point of our analysis compared to previous approaches in literature is that we directly work in a fully discrete setting and therefore avoid an additional projection error. Moreover, we do not need an a priori coupling of the block rank and the mesh width.

References

[1] M. Faustmann, J. M. Melenk, D. Praetorius: A new proof for existence of $\mathcal{H}$-matrix approximants to the inverse of FEM matrices: the Dirichlet problem for the Laplacian, *ASC Report 51/2012*.

[2] M. Faustmann, J. M. Melenk, D. Praetorius: Existence of $\mathcal{H}$-matrix approximants to the inverse of BEM matrices, *work in progress*.

Strongly scalable numerical methods to simulate cardiovascluar tissues

Christoph Augustin[1], Olaf Steinbach[2], Gernot Plank[1]

[1] Medical University of Graz
[2] Graz University of Technology

Anatomically realistic and biophysically detailed multiscale computer models of cardiovascular tissues like the heart or arterial vessels are playing an increasingly important role in advancing our understanding of integrated cardiac function in health and disease. However, such detailed multiphysics simulations are computationally vastly demanding. While current trends in high performance computing (HPC) hardware promise to alleviate this problem, exploiting the potential of such architectures remains challenging. Strongly scalable algorithms are necessitated to achieve a sufficient reduction in execution time by engaging a large number of cores. This imposes many constraints on design and implementation of solver codes.

We discuss two different parallel approaches, the finite element tearing and interconnecting (FETI) method and a proper domain decomposition algebraic multigrid, to solve the non-linear elasticity problems arising from the simulation of the mechanical behavior of cardiovascular tissues. Scalability results for these mechanical simulations will be presented. We will also show first results of weakly and strongly coupled electromechanical problems and discuss challenges that need to address with regard to highly scalable parallel implementations.

Hybrid cG/dG Finite Element Methods for Solving Semilinear Parabolic Equations

Elias Karabelas[1], Olaf Steinbach[1]

[1]Graz University of Technology

In this talk we discuss hybrid cG/dG finite element methods which are continuous in space and discontinuous in time for the solution of semilinear parabolic partial differential equations. We present a stability and error analysis and we give some numerical results. This cG/dG approach allows rather general discretizations in space and time as well as adaptive refinement strategies. One application in mind is the simulation of the cardiac bidomain equations.

Two-scale homogenization of the eddy-current problem in laminated media

Antti Hannukainen[1], Karl Hollaus[2], Joachim Schöberl[2]

[1] Aalto University
[2] TU Vienna

To reduce losses, critical electrical machine parts are constructed by laminating thin insulator coated iron sheets together. Since iron and insulator have very different electromagnetic properties, the permeability μ and the conductivity σ oscillate rapidly in the laminated parts. This oscillating behavior leads to difficulties in the numerical simulation of eddy current losses occurring in the machine.

In time-harmonic analysis, the eddy current losses are computed by solving the eddy current problem: find the vector potential A such that

$$\mathrm{i}\omega\sigma A + \nabla \times \mu^{-1} \nabla \times A = J \text{ and } \nabla \cdot A = 0, \tag{1}$$

inside the machine. Solving this problem in the laminated region with sufficient accuracy requires the use of finite element mesh finer than the thickness of the individual sheets. As laminations are very thin compared to the size of the machine, this requirement leads to unrealistically fine discretizations and too high computational costs.

A simple and often used engineering strategy for avoiding numerical problems in the laminated parts is to assume that the laminate is non-conducting perpendicular to the sheets and to use an effective permittivity. These assumptions eliminate the oscillations of μ and σ that cause difficulties for numerical schemes. However, this comes at the price of neglecting all current loops perpendicular to the laminated sheets. Due to this, the losses computed using such strategy cannot accurately approximate the real losses.

In this talk, we consider using two-scale homogenization to model the laminated parts. Homogenization allows the losses to be accurately computed using a coarse computational grid with a reasonable computational cost. We will present the derivation of the homogenized problem and numerically study the error induced in the homogenization procedure. This talk is a continuation of our earlier work on the topic, see [1, 2].

References

[1] K. Hollaus and J. Schöberl. Homogenization of the Eddy Current Problem in 2D, In Proceedings of the 14th International IGTE Symposium on Numerical Field Calculation in Electrical Engineering (2010), 154–159.

[2] K. Hollaus and J. Schöberl. A Linear FEM Benchmark for the Homogenization of the Eddy Currents in Laminated Media in 3D, to appear in IFAC (2013).

An Efficient Robust Solver for Optimal Control Problems for the Stokes Equations in the Time-Harmonic Case

Wolfgang Krendl[1], Valeria Simoncini[2], Walter Zuhlehner[1]

[1]Johannes Kepler University
[2] Università di Bologna

In this talk we will construct a robust solver for the optimal control problem for the Stokes equation, in the time-harmonic case. The discretization of the corresponding optimality system leads to a large and sparse 8x8 block matrix in saddle point form. We use an iterative solver, more precisely, we apply the MINRES method. To gurantee efficiency, we constructed a preconditioner for the MINRES-method, which is robust with respect to the mesh size, the frequency ω and the control parameter α. Numerical examples are given which illustrate the theoretical results.

A High Order Discontinuous Galerkin Method for the Boltzmann equation

Gerhard Kitzler[1], Joachim Schöberl[2]

[1] Vienna UT
[2] Vienna UT

The Boltzmann equation is a statistical model for gases. Its solution function $f(t, x, v)$ is usually called density distribution function and describes the average number of particles having a position close to x, and a velocity close to v at time t. Boltzmanns equation governs the time evolution of this distribution function:

$$\frac{\partial f}{\partial t} + \mathrm{div}_x(v\, f) = Q(f) \tag{1}$$

The collision operator on the right hand side (1) describes the effect of binary collisions within the particles. It is local in time and position but global in velocity.
For a discretication in the momentum domain, the solution f is expanded as

$$f_M = e^{-\frac{|v-V(x,t)|^2}{T(x,t)}} \sum_{m=0}^{n} c_m\, L_m\left(\frac{v-V(x,t)}{\sqrt{T(x,t)}}\right), \tag{2}$$

with multivariate Lagrange polynomials L_m, in Gauss Hermite collocation points. V and T are the macroscopic unknown quantities velocity and temperature of the gas.
In the space domain we use a high order discontinuous galerkin method with natural upwind fluxes for discretization.

Exponential integrators for shallow water equations

Petra Csomós[1], Alexander Ostermann[1]

[1] University of Innsbruck

Shallow water equations not only play an important role in hydrodynamic simulations, but also their numerical treatment can be successfully investigated. In the present talk we study the application of exponential integrators for three dimensional inviscid shallow water equations with rotation terms. First, we discuss the derivation and the linearisation of these equations. Then we show how to formulate them as an abstract semilinear problem for which the numerical solution can be obtained by exponential integrators. We study the convergence of the exponential Euler method in this case and discuss the benefits of using higher order methods. We also present some numerical experiments.

A perturbation result for quasi-linear stochastic differential equations in UMD Banach spaces

Erika Hausenblas[1], Sonja Cox[2]

[1] Montanuniversity Leoben, Austria
[2] ETH-Zürich, Switzerland

In a joint work with Cox (see [1, 2]) we have shown a perturbation result with respect to a stochastic evolution equation. To be more precise, we consider the effect of perturbations of A on the solution to the following quasi-linear parabolic stochastic partial differential equation:

$$\begin{cases} dU(t) & = AU(t)\,dt + F(t,U(t))\,dt + G(t,U(t))\,dW_H(t), \quad t>0; \\ U(0) & = x_0. \end{cases} \tag{1}$$

Here A is the generator of an analytic C_0-semigroup on a UMD Banach space X with type τ, $G:[0,T]\times X \to \mathcal{L}(H, X^A_{\theta_G})$ and $F:[0,T]\times X \to X^A_{\theta_F}$ for some $\theta_G > -\frac{1}{2}$, $\theta_F > -\frac{3}{2}+\frac{1}{\tau}$. We assume F and G to satisfy certain global Lipschitz and linear growth conditions. The spaces $X^A_{\theta_F}$ and $X^A_{\theta_G}$ are certain interpolation, resp. extrapolation spaces.

Let A_0 denote the perturbed operator and U_0 the solution to (1) with A substituted by A_0. We provide estimates for $\|U-U_0\|_{L^p(\Omega;C([0,T];X))}$ in terms of $D_\delta(A,A_0) := \|R(\lambda : A) - R(\lambda : A_0)\|_{\mathcal{L}(X^A_{\delta-1},X)}$. Here $\delta\in[0,1]$ is assumed to satisfy $0\le\delta<\min\{\frac{3}{2}-\frac{1}{\tau}+\theta_F, \frac{1}{2}-\frac{1}{p}+\theta_G\}$.

With the help of this result, we prove almost sure uniform convergence rates for space approximations of semi-linear stochastic evolution equations with multiplicative noise in Banach spaces. The space approximations we consider are spectral Galerkin and finite elements, but can also be applied to wavelets in Besov spaces. A more theoretical application is the Yosida approximation, to which the result can be applied as well.

References

[1] S. Cox, and E. Hausenblas. *Pathwise space approximations of semi-linear parabolic SPDEs with multiplicative noise. International Journal of Computer Mathematics*, 89, no. 18, 2460–2478, 2012.

[2] S. Cox, and E. Hausenblas. *A perturbation result for quasi-linear stochastic differential equations in UMD Banach spaces*, submitted, 20 pages, April 2012.

Numerical aspects of audio processing

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Sound signals play a central role in human life and the manner sound is perceived is highly sophisticated, complex and context-dependent. Since the amount of sound data that are automatically stored, searched and processed, grows dramatically, there is also a growing need for understanding the inherent structures of sound and their implications for human listeners.

Ideally, an analysis tool should be able to render a representation that allows for visual display reflecting a user's acoustical impression; such a representation turns out to be beneficial for processing in the sense of perceptual results. Often, this kind of representation can be achieved by applying adaptive time-frequency representations. These representations are intuitively appealing, but often computationally inefficient.

In this talk we will describe the design of adaptive representations based on *frame theory* and we will present a framework which allows for FFT-based analysis and FFT-based perfect or approximate reconstruction. Examples for this kind of framework can be found in [1].

References

[1] N. Holighaus, M. Dörfler, G. Velasco, and T. Grill. **A framework for invertible, real-time constant-Q transforms.** *IEEE Transactions on Audio, Speech and Language Processing*, to appear, 2012.

Numerical Questions in Time-frequency Analysis

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By now Gabor analysis is a mature subfield of time-frequency analysis. It is well understood that for any lattice Λ in the TF-plane the corresponding Gabor family $(\pi(\lambda)g)$ is a Gabor frame with good properties if the Gabor atom g is a nice function, because the commutation properties of the frame operator S imply that also the dual atom $\tilde{g}$ has good properties.

Parallel with an improved understanding of the functional analytic properties of these non-orthogonal expansions (which in addition have some built in redundancy, but also robustness) there are more and more efficient methods which allow to provide in a fast way the minimal norm coefficients for a given signal in an efficient way.

We plan to provide some insight into the questions, methods and algorithms arising in this context. The demonstrations are all performed in a MATLAB environment. Corresponding MATLAB files can be found on the NuHAG web-page (`www.nuhag.eu`), including the LTFAT toolbox (Linear Time-Frequency Toolbox), developed by Peter Soendergaard (presently at ARI = Acoustic Research Institute, OEAW, Vienna).

Local error structures of higher-order exponential splitting schemes

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A multistage (s-stage) splitting approximation step applied to a linear evolution equation $\frac{d}{dt}\,u = Hu = Au + Bu$ has the general form

$$\mathcal{S}(t)u = e^{tB_s}\,e^{tA_s}\,\cdots\,e^{tB_1}\,e^{tA_1}u \approx e^{t(A+B)}u, \tag{1}$$

where t is the time step, $A_j = a_j A$, $B_j = b_j B$, and a_j, b_j are real or complex coefficients to be chosen in a way such that a certain approximation order is realized.

A systematic way of studying the error structure of splitting schemes is based on an integral representation for the local error in terms of the defect

$$\mathcal{D}(t)u = \frac{d}{dt}\,\mathcal{S}(t)u - H\mathcal{S}(t)u. \tag{2}$$

This way of representing the error differs from existing approaches which are more or less based on BCH or related formulas. We give an exposition of this approach and discuss order conditions, the combinatorical structure of the defect, and the design and analysis of a posteriori error estimators which are obtained by approximating the local error by a computable defect-based estimate.

A general integrator for the Landau-Lifshitz-Gilbert equation

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The understanding of magnetization dynamics, especially on a microscale, is of utter relevance, for example in the development of magnetic sensors, recording heads, and magneto-resistive storage devices. In the literature, it is well-accepted that dynamic micromagnetic phenomena are modeled best by the Landau-Lifshitz-Gilbert equation (LLG) which describes the behaviour of the magnetization under the influence of some effective field that may consist of several contributions such as the microcrystalline anisotropy or the demagnetization field. Numerical challenges for the time integration arise from strong nonlinearities, a non-convex modulus constraint and possible non-local field contributions.

Recently there has been a huge progress in the mathematical literature for weak solvers to LLG. In [Alouges, (2008)], the author introduced an integrator that requires to solve only one linear system per timestep and still guarantees unconditional convergence towards a weak solution of LLG. While this work was done for an effective field with exchange energy only, in [Alouges (2011)] and [Goldenits (2012)], the analysis was extended to cover a more general, however, linear effective field.

In our contribution, we extend the above approach to show the full potential of this solver. By exploiting an abstract framework, we can cover general field contributions that might be nonlinear, non-local, and/or time-dependent. Applications include multiscale modeling, coupling of LLG to the full Maxwell's equations, or even to the conservation of momentum equation to include magnetostrictive effects. Even in this general setting, we can still prove unconditional convergence while sustaining very little computational effort.

Boundary control of exterior boundary value problems

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In this presentation we discuss boundary control problems subject to second order partial differential equations in unbounded exterior domains. Examples involve the Laplace and Helmholtz equations. Since the control is considered in $H^{\frac{1}{2}}(\Gamma)$ the regularisation is realised by the exterior Steklov-Poincaré operator. For the numerical approximation we consider a symmetric Galerkin boundary element method and we apply a semi-smooth Newton method in the case of box constraints. Some numerical examples are given.

References

[1] G. Of, T. X. Phan, O. Steinbach: Boundary element methods for Dirichlet boundary control problems. Math. Methods Appl. Sci., 33(18) (2010), 2187-2205.

[2] G. Of, T. X. Phan, O. Steinbach: An energy space finite element approach for elliptic Dirichlet boundary control problems. Graz University of Technology Berichte aus dem Institut für Numerische Mathematik, Bericht 2012/5.

[3] T. X. Phan. Boundary element methods for boundary control problems. PhD thesis Graz University of Technology, (2011).

New regularity results and improved error estimates for optimal control problems with state constraints

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In this talk we consider optimal control problems governed by an elliptic equation and subject to pointwise state constraints. The analysis as well as the numerical analysis of such problems is difficult due to the lack of regularity of the Lagrange multiplier as well as of the adjoint state. We provide a new regularity result for the adjoint state, which allows to improve the a priori error estimates for the finite element discretization of the problem.

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List of Workshops

1. Austrian Numerical Analysis Day , 13.4.2005,
   Universitätszentrum Obergurgl (Universität Innsbruck)
2. Austrian Numerical Analysis Day , 27.-28.4.2006, TU Graz
3. Austrian Numerical Analysis Day, 26.-27.4.2007, TU Wien
4. Austrian Numerical Analysis Day , 24.-25.4.2008, Universität Linz
5. Austrian Numerical Analysis Day , 7.-8.5.2009, Universität Innsbruck
6. Austrian Numerical Analysis Day, 6.-7.5.2010, Universität Salzburg
7. Austrian Numerical Analysis Day, 5.-6.5.2011, Universität Klagenfurt
8. Austrian Numerical Analysis Day, 10.-11.5.2012, TU Wien
9. Austrian Numerical Analysis Day, 11.-12.4.2013, TU Graz
10. Austrian Numerical Analysis Day, 7.–9.7.2014, Universität Wien

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2012/2 O. Steinbach: Boundary element methods in linear elasticity: Can we avoid the symmetric formulation?

2012/3 W. Lemster, G. Lube, G. Of, O. Steinbach: Analysis of a kinematic dynamo model with FEM–BEM coupling.

2012/4 O. Steinbach: Boundary element methods for variational inequalities.

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2012/12 T. X. Phan, O. Steinbach: Boundary element methods for parabolic boundary control problems.

2013/1 L. John, P. Pustějovská, O. Steinbach: On hemodynamic indicators related to aneurysm blood flow.