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Weak and strong convergence of a BDF2-type integrator for the Landau–Lifshitz–Gilbert equation in micromagnetics

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We consider the Landau–Lifshitz–Gilbert (LLG) equation, a nonlinear partial differential equation that models time-dependent micromagnetic phenomena. We propose a full discretization scheme based on first-order finite elements in space and an implicit BDF2-type two-step method in time. Despite the nonlinearities of LLG, only one linear system must be solved per time-step. This scheme is, to our knowledge, the first BDF2-type integrator for LLG that guarantees both unconditional weak convergence to weak solutions of LLG, as well as optimal strong convergence in the presence of smooth solutions. Specifically, we prove a-priori error estimates of order $\mathcal{O}(h + k^2)$, where k and h denote the time-step size and spatial mesh-size, respectively – that is, second-order in time and first-order in space. Numerical experiments illustrate and confirm the theoretical results.

References

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