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# An Optimal Control Problem Approach for the Resolution of the Inverse Poisson Problem

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We consider the problem of localizing and identifying a target set of acoustic sources. Methods already exist but they are challenged as soon as there are multiple sources in the domain of interest, if there are too many sources of noises, or if there is too little microphones available. Classical multi-source localization and tracking methods typically operate in the frequency domain and leverage spectral differences between sources, which can limit their robustness in noisy environments or when multiple sources have similar spectral content. This motivates the search for alternative approaches that rely on weaker assumptions.

In this work, we consider an Optimal Control Problem (OCP) approach with Partial Differential Equation (PDE) constraint[1] to evaluate the positions of our sources over time. The OCP formulation has shown its usefulness in boundary control problems[2]. Approaches based on boundary control yield accurate results for the inverse problem associated to the Poisson and Helmholtz equation[3], in particular for low wavenumbers. We are interested in the minimal number of microphones required to produce an accurate description of the positions of target source term without prior knowledge of the source. Considering a limited set of microphones, we plan to reconstruct the positions of the sources using fast, real-time numerical tools to dynamically track static targets first, then extend our work to moving targets.

## References

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