

Austrian Numerical Analysis Day 2026

Computer-assisted Proofs of Reachability Analysis for Linear Control Systems under Bounded Constraints

Ivan HASENOHR¹, Camille POUCHOL², Yannick PRIVAT³, Christophe ZHANG⁴

¹ Alpen-Adria Universität, Klagenfurt ² MAP5, Université Paris Cité, France

³ IUF & Université de Lorraine, France

⁴ Inria Paris & LJLL, Sorbonne Université, France

Let us consider a linear control problem of the form

$$\begin{cases} \partial_t y(t) = Ay(t) + Bu(t) & \forall t \in (0, T), \\ y(0) = y_0 \in X, \\ u(t) \in \mathcal{U} \subset U & \forall t \in (0, T), \end{cases}$$

where X, U are (possibly infinite-dimensional) Hilbert spaces and $\mathcal{U} \subset U$ is a convex, closed and bounded constraint set. Considering a target y_f , it follows from a separation argument that y_f is not reachable in time T if and only if

$$\exists p_f \in X, \quad J(p_f) := \int_0^T \sigma_{\mathcal{B}U}(S_t^* p_f(t)) dt - \langle y_f, p_f \rangle + \langle y_0, S_T^* p_f \rangle < 0,$$

where $(S_t)_t$ is the semigroup associated to A , and $\sigma_{\mathcal{U}}$ is the support function of the constraint set. This formulation will prove to be especially adapted to a numerical discretisation aiming for rigorous numerics.

Under standard assumptions on the operators (A continuous and coercive on $V \hookrightarrow X$, B bounded) and a-priori assumptions on the discretisation space $V_h \subset V$, we introduce a space-time discretisation of J , allowing us to obtain the following bound:

$$|J(p_f) - J_{\Delta t, h}(p_{fh})| \leq C(\Delta t + h^2)(\|A^* p_f\|_X + \|p_f - p_{fh}\|_V) \quad \forall p_f, p_{fh} \in \mathcal{D}(A^*) \times V_h,$$

where C is an explicit constant depending on various problem-related parameters. This upper-bound allows for computer-assisted proofs in the following meaning: should one find $p_{fh} \in V_h$ and associate it $p_f \in \mathcal{D}(A^*)$ such that

$$J_{\Delta t, h}(p_{fh}) + |J(p_f) - J_{\Delta t, h}(p_{fh})| + e_r(p_{fh}) < 0,$$

where $e_r(p_{fh})$ is an upper-bound of the rounding errors of the numerical computation of $J_{\Delta t, h}(p_{fh})$, then $J(p_f) < 0$ and hence y_f is not reachable in time T under constraints \mathcal{U} .

This methodology is then applied to prove the non-reachability of various 1D-heat-equation based reachability problems. These results have yielded the preprint [2], which is a continuation of the finite-dimensional paper [1].

References

- [1] HASENOHR Ivan, POUCHOL Camille, PRIVAT Yannick and ZHANG Christophe: Computer-assisted proofs of non-reachability for linear finite-dimensional control systems. *SIAM Journal on Control and Optimization*, 2025, vol. 63.5, 3272-3296.
- [2] HASENOHR Ivan, POUCHOL Camille, PRIVAT Yannick and ZHANG Christophe: Computer-assisted proofs of non-reachability for linear parabolic PDEs under bounded control constraints. Preprint 2025.