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Adaptive finite element methods with optimally preconditioned GMRES

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This talk addresses optimal complexity of adaptive finite element methods (AFEMs) for general second-order linear elliptic partial differential equations (PDEs) in the Lax–Milgram setting. We present recent work [1] that formulates an adaptive algorithm which steers the local mesh-refinement as well as the termination of a generalized minimal residual solver (GMRES) with optimal preconditioner to solve the arising non-symmetric finite element systems. It is shown that the algorithmic interplay of mesh-refinement and iterative solver is optimal: A fully computable quasi-error monitoring discretization error and algebraic solver error guarantees unconditional convergence for any choice of adaptivity parameters, i.e., the algorithm cannot fail to convergence. Moreover, the quasi-error even decays with optimal rates with respect to the overall computational complexity if the adaptivity parameters are chosen sufficiently small. As a novel contribution, the algorithm includes an adaptive feedback-control for the solver-termination parameter that monitors and ensures full R-linear convergence.

References

[1] T. Führer, P. Hilbert, A. Miraçi, D. Praetorius: *Adaptive finite element methods with optimally preconditioned GMRES guarantee optimal complexity.* in preparation (2026).