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Comparison of iterative methods for discretized nonsymmetric elliptic problems

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In this talk, we compare the performance of the Conjugate Gradient method applied to the normal equation (CGN) and the Generalized Conjugate Residual method (GCR) when solving a system of linear equations obtained from the finite element discretization of nonsymmetric elliptic problems, such as stationary convection-diffusion models of flows. Similar comparative studies have been carried out by earlier authors in [1], where it was shown that for general matrices there is no single "best" nonsymmetric iterative method that we know of, which would dominate all others in terms of the number of iterations required for convergence. However, our numerical experiments indicate that for the particular class of problems considered here, it is possible to predict which of the two methods will converge faster depending on the coefficients of the PDE.

After presenting the motivating numerical results, we justify these observations by comparing the linear convergence estimates of the two iterative methods. Based on this theoretical comparison, we derive explicit upper and lower bounds on the size of the coefficients of the PDE that guarantee the dominance of the CGN method, and we also explain the characteristic shapes of the curves observed in the comparative numerical tests. In addition to the standard finite element method (FEM), we also present results for its streamline diffusion variant (SDFEM), which is particularly relevant for convection-dominated problems arising in physical applications.

To demonstrate the broader applicability of our results, we conclude the talk with an analysis of a convection-dominated meteorological problem having real wind speed data as coefficients.

References

[1] N. M. Nachtigal, S. C. Reddy, L. N. Trefethen: How Fast are Nonsymmetric Matrix Iterations?. SIAM J. Matrix Anal. Appl., 13 (1992), 778-795.