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Analysis of the Generalized Shape Operator

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In many applications, including shell models and geometric flows, surface curvature plays a central role. Discrete differential geometry (DDG) approximates curvature-related quantities on discretized surfaces. Embedding DDG into a finite element framework makes rigorous numerical analysis tools available, both for the study of existing DDG constructions and for their systematic extension. In this context, so-called distributional finite elements are essential. They allow for weak regularity, so differential operators must be interpreted in the sense of distributions.

In this talk, we study the approximation of the shape operator on surfaces whose discrete geometry is described by Lagrange finite elements. Our approach combines the DDG concept of dihedral angles with Hellan–Herrmann–Johnson finite elements [1]. Based on a novel integral representation of the error, we prove rigorous error estimates [2]. Under mild assumptions, we establish convergence in the negative Sobolev norm H^{-1}, thereby showing that the dihedral-angle-based DDG approximation always converges. Under stronger assumptions, we obtain optimal L^2-convergence for the discrete L^2-Riesz representative of the generalized shape operator. Finally, we discuss applications to nonlinear shell models [3] and present numerical examples using the finite element library NGSolve (www.ngsolve.org).

References

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