

Austrian Numerical Analysis Day 2026

An Elliptic Space-Time Variational Formulation for Parabolic Problems

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In this talk we consider parabolic problems in full space-time. We start by discussing the ordinary differential equation

$$\partial_t u(t) + \mu u(t) = f(t), \quad t \in (0, T), \quad u(0) = u_0, \quad (1)$$

where $\mu > 0$ is a surrogate for the spatial operator. We derive a systematic approach to obtain elliptic space-time variational formulations for this type of equation using the concept of the polar decomposition of operators in Hilbert spaces. Afterward, we illustrate discretization and preconditioning strategies, still for the ODE case. These results can then be directly transferred to the parabolic PDE case, which we illustrate on the example of the heat equation.

References

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