Austrian Numerical Analysis Day 2026

Hybridizable discontinuous Galerkin methods for the wave equation on beam network models

Lucia Swoboda[1], Moritz Hauck[2], Joseph Holten[2], Axel Målqvist[1], Andreas Rupp[3]

[1] Chalmers University of Technology and University of Gothenburg, Sweden
[2] Karlsruhe Institute of Technology, Germany
[3] Saarland University, Saarbrücken, Germany

Network models are used to describe complex structures found in fiber-based materials such as paper, or biological tissues. In these models, fibers are represented as edges connected at nodes, and are modeled as Timoshenko beams to capture the mechanical behavior of the material. Understanding wave propagation in such networks is important for predicting material response and improving performance in applications such as papermaking.

We present a hybridizable discontinuous Galerkin (HDG) method for the spatial discretization of the wave equation on fiber networks, combined with a θ-scheme for time integration. Through hybridization, the problem is reformulated as a symmetric positive definite system on the network nodes. The HDG spatial discretization achieves arbitrary-order convergence under mesh refinement without increasing the size of the global system. The approach builds on work in [1] and [2]. We establish convergence and error estimates, supported by numerical experiments.

References

[1] B. Cockburn, J. Gopalakrishnan, and F.-J. Sayas: A projection-based error analysis of HDG methods. Math. Comp., 79 (2010), 1351–1367.

[2] M. Hauck, A. Målqvist, and A. Rupp: Arbitrary order approximations at constant cost for Timoshenko beam network models. ESAIM: Mathematical Modelling and Numerical Analysis, (2025).