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A Robust and Time-Parallel Preconditioner for Parabolic Least Squares Problems

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We discuss preconditioners for space-time formulations of parabolic evolution equations. As model problem, we consider the heat equation. We derive the space-time variational problem to be solved from a least-squares minimization problem. While this approach potentially increases the condition number, it leads to an elliptic problem, which allows for simpler preconditioners and which leads to a system that can be solved using a conjugate gradient solver that tends to converge faster than a minimum residual solver. In order to formulate the least-squares problem, one has to choose a norm to measure the residual; we consider both a L^2-norm over the whole space-time cylinder and the Bochner space norm of functions that are L^2 in time and H^{-1} in space. For both choices, we propose an operator preconditioner based on the mapping properties of the underlying differential operator. In order to discretize the problem, we use Isogeometric Analysis since it allows for an effortless construction of sufficiently smooth basis functions.

To realize the preconditioner, one has to solve a problem over the whole space-time cylinder that is elliptic with respect to suitable anisotropic norms. A fast diagonalization approach in time can be used to reformulate the prob- lem as a collection of elliptic problems in space only. These problems are not only smaller, they can also be solved concurrently.

We show the efficiency of the preconditioner by rigorous analysis and illustrate it with numerical experiments.