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Matching triangular and quadrilateral C^1 elements

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We present a family of triangular and quadrilateral finite elements for degrees $p \geq 3$ that can be matched C^1. For all degrees $p \geq 5$ the elements are polynomial and are formed by the Argyris elements, cf. [1], and their quadrilateral counterparts, developed by Brenner & Sung in [2]. The coupling of such elements is described in [3]. Quadrilateral C^1 elements for $p = 3$ and $p = 4$ were introduced and analyzed in [4]. They are macro-element constructions that require regular splits into 3×3 and 2×2 sub-elements, respectively, which derive from tensor-product spline constructions. Until recently, no matching triangular elements were known.

It turns out that the bi-cubic quadrilateral element can be C^1-matched with the cubic, triangular C^2 element on the Wang–Shi 3-split, described in [5]. Moreover, a suitable quartic triangular C^3 element on the Wang–Shi 2-split, i.e., the Powell–Sabin 12-split, can be constructed to C^1-match with the bi-quartic C^1 quadrilateral element. We show how these elements are constructed and present some applications. The presented results are based on ongoing research developed in collaboration with A. Bressan and J. Grošelj.

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References

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