ADAPTIVE FAST BOUNDARY ELEMENT METHODS IN INDUSTRIAL APPLICATIONS
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# A Direct Boundary Integral Equation Method for Mixed Dielectric-PEC Scatterers

Marie Curie Fellowships

# Rationale

- Electromagnetic Compatibility (EMC) problems (asserting certain field values within enclosures) $\Rightarrow$ Transmission problems (coupling through apertures):
  - Apertures in conducting screen;
  - Waveguide-to-cavity coupling, Cavity-to-cavity coupling, etc;
- Most solutions are:
  - Problem specific (Spherical / cylindrical /parallelepiped  geometry);
  - Problem approximations (infinite metallic bodies);
- EFIE + Equivalence principle largely used
  - Limitations: EFIE does not treat $\varepsilon_{int} \neq \varepsilon_{ext}$;
  - Complex approach, often impractical for real-life geometries;
- EFIE & CFIE: big difficulties in accommodating combined PEC and Transmission boundary conditions

# Our Approach

- Direct Boundary Integral Method : The unknowns correspond to physical tangential components of electric and magnetic field on surface of scatterer $\Rightarrow$ same quantities to occur in transmission conditions;
- Use electric-to-magnetic (or Dirichlet-to-Neuman) mapping operators;
- Accommodate naturally PEC and Transmission boundary conditions.
- Structure of discretized equation perfectly matches symmetry of coupled scattering problem;
- Galerkin discretization scheme by means of divergence conforming vectorial functions (RWG – functions);
- No specific geometry assumed / No simplifying assumptions;
- Treats $\varepsilon_{int} \neq \varepsilon_{ext}$ and/or $\mu_{\text{int}} \neq \mu_{ext}$

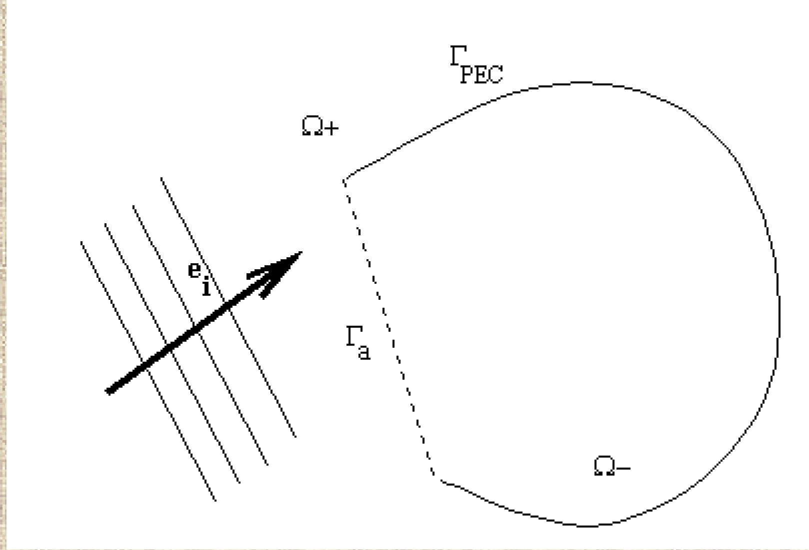
# Transmission Problem. Traces

**Electric** (Dirichlet) trace:

$$\gamma_t \mathbf{e} := \mathbf{e} \times \mathbf{n}$$

**Transmission problem** defined as:

$$\left.\begin{aligned}
&\operatorname{curl}\mathbf{e} = -j\omega\mu\,\mathbf{h} \qquad \operatorname{curl}\mathbf{h} = j\omega\varepsilon\,\mathbf{e} \;\text{ in } \Omega^+ \cup \Omega^- \\
&\gamma_t^+\mathbf{e} = 0 \quad\text{and}\quad \gamma_t^-\mathbf{e} = 0 \quad \text{on } \Gamma_{PEC} \\
&\gamma_t^-\mathbf{e} - \gamma_t^+\mathbf{e} = \gamma_t^+\mathbf{e}_i \qquad \text{on } \Gamma_a \\
&\gamma_t^-\mathbf{h} - \gamma_t^+\mathbf{h} = \gamma_t^+\mathbf{h}_i \qquad \text{on } \Gamma_a \\
&\lim_{|\mathbf{x}|\to\infty}\left(\sqrt{\mu}\,\mathbf{h}^S \times \frac{\mathbf{x}}{|\mathbf{x}|} - \sqrt{\varepsilon}\mathbf{e}^S\right) = 0 \\
&k_\pm = \omega\sqrt{\varepsilon_\pm \mu_\pm}
\end{aligned}\right\}$$




$$\operatorname{curl}\operatorname{curl}\mathbf{e} - k_\pm^2\mathbf{e} = 0 \;\text{ in }\; \Omega^+ \cup \Omega^-$$

$$\gamma_t^-\mathbf{e} - \gamma_t^+\mathbf{e} = \gamma_t^+\mathbf{e}_i \qquad \text{on } \Gamma_a$$

$$\frac{k^-}{\mu^-}\gamma_N^-\mathbf{e} - \frac{k^+}{\mu_0}\gamma_N^+\mathbf{e} = \frac{k^+}{\mu_0}\gamma_N^+\mathbf{e} = \gamma_t^+\mathbf{h}_i \quad \text{on } \Gamma_a$$

**Magnetic** (Neuman) trace:

$$\gamma_N \mathbf{e} := \frac{1}{k}\gamma_t \circ \operatorname{curl}(\mathbf{e})$$

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# Framework of Function Spaces

Electric wave equations have solutions in:

$$\boldsymbol{H}(\mathbf{curl},\Omega_s) := \left\{ \mathbf{u} \in \left(L^2(\Omega_s)\right)^3 \mid \nabla\times\mathbf{u} \in \left(L^2(\Omega_s)\right)^3 \right\}$$

$\boldsymbol{H}_{\Gamma_{PEC}}(\mathbf{curl},\Omega_s) := \left\{ \mathbf{u} \in \boldsymbol{H}(\mathbf{curl},\Omega_s) \mid \gamma_t^- \mathbf{u}\big|_{\Gamma_0} = 0 \right\}$ a closed subspace of $\boldsymbol{H}(\mathbf{curl},\Omega_s)$

**Trace** theorem for $\boldsymbol{H}(\mathbf{curl},\Omega_s)$: $(\exists)\ \mathbf{H}_x^{-1/2}(\mathrm{div}_\Gamma,\Gamma)$ such that:

$$\gamma_t^- : \mathbf{H}_{\Gamma_{PEC}}\left(\mathrm{curl},\Omega^-\right) \mapsto \mathbf{H}_x^{-1/2}(\mathrm{div}_\Gamma,\Gamma) \text{ and } \gamma_t^+ : \mathbf{H}_{\Gamma_{PEC}}\left(\mathrm{curl},\Omega^+\right) \mapsto \mathbf{H}_x^{-1/2}(\mathrm{div}_\Gamma,\Gamma)$$

are continuous and surjective mappings

Based on bilinear **anti-symmetric** pairing: $\langle \mathbf{u},\mathbf{v}\rangle_{\tau,\Gamma} = \int_\Gamma (\mathbf{u}\times\mathbf{n})\cdot\mathbf{v}\,dS \quad \mathbf{u},\mathbf{v} \in L_t^2(\Gamma)$

$\mathbf{H}_x^{-1/2}(\mathrm{div}_\Gamma,\Gamma)$ becomes self-dual.

Furthermore, for:

$$\boldsymbol{H}_x^{-1/2}(div_\Gamma,\Gamma_a) := \left\{ \phi \in \boldsymbol{H}_{x,00}^{-1/2}(div_\Gamma,\Gamma_a), \tilde{\phi} \in \boldsymbol{H}_x^{-1/2}(div_\Gamma,\Gamma) \right\}$$

$\gamma_t^- : \boldsymbol{H}_{\Gamma_0}(\mathbf{curl},\Omega_s) \mapsto \boldsymbol{H}_x^{-1/2}(\mathrm{div}_\Gamma,\Gamma_a)$ Is also a continuous and surjective mapping

# Maxwell Poincaré-Steklov Operators

**Exterior** problem

$$\operatorname{curl}\operatorname{curl}\mathbf{e} - k_+^2\mathbf{e} = 0 \quad \text{in} \quad \Omega^+$$

$\gamma_t^+\mathbf{e} = 0$ on $\Gamma_{PEC}$

$\gamma_t^+\mathbf{e} = \zeta$ on $\Gamma_a$

$$\mathbf{e}_T = \mathbf{e} + \mathbf{e}_i$$

Silver-Mueller Radiation condition

$$\mathbf{T}^+ : \begin{cases} \boldsymbol{H}_x^{-1/2}(\operatorname{div}_\Gamma, \Gamma_a) \mapsto \boldsymbol{H}_x^{-1/2}(\operatorname{div}_\Gamma, \Gamma) \\ \zeta \mapsto \gamma_N^+\mathbf{e} \end{cases}$$

**Interior** problem

$$\operatorname{curl}\operatorname{curl}\mathbf{e} - k_-^2\mathbf{e} = 0 \quad \text{in} \quad \Omega^-$$

$\gamma_t^-\mathbf{e} = 0$ on $\Gamma_{PEC}$

$\gamma_t^-\mathbf{e} = \zeta$ on $\Gamma_a$

$$\mathbf{T}^- : \begin{cases} \boldsymbol{H}_x^{-1/2}(\operatorname{div}_\Gamma, \Gamma_a) \mapsto \boldsymbol{H}_x^{-1/2}(\operatorname{div}_\Gamma, \Gamma) \\ \zeta \mapsto \gamma_N^-\mathbf{e} \end{cases}$$

$$\mathbf{T}^-\gamma_t^-\mathbf{e} - \mathbf{T}^+\gamma_t^+\mathbf{e} = \frac{k^+}{\mu_0}\gamma_N^+\mathbf{e} = \gamma_t^+\mathbf{h}_i$$

Set on $\mathbf{H}_{x,00}^{-1/2}(\operatorname{div}_\Gamma, \Gamma_a)$ since $\zeta = 0$ on $\partial\Gamma_a$

Dual space $\mathbf{H}_x^{-1/2}(\operatorname{div}_\Gamma, \Gamma_a)$ provide appropriate test functions.

# Stratton-Chu Integral Representations

For **Interior/ Exterior** problem, Stratton-Chu representations:

$$\mathbf{e} = \mathbf{\Psi}_{DL}^{k}\left(\gamma_t^- \mathbf{e}\right) + \mathbf{\Psi}_{SL}^{k}\left(\gamma_N^- \mathbf{e}\right) \qquad \text{in } \boldsymbol{H}\left(\text{curl}^2, \Omega^-\right)$$

$$\mathbf{e} = -\mathbf{\Psi}_{DL}^{k}\left(\gamma_t^+ \mathbf{e}\right) - \mathbf{\Psi}_{SL}^{k}\left(\gamma_N^+ \mathbf{e}\right) \qquad \text{in } \boldsymbol{H}\left(\text{curl}^2, \Omega^+\right) \tag{1}$$

**Maxwell single** and **double layer potentials**::

$$\mathbf{\Psi}_{SL}^{k}(\mathbf{e})(\mathbf{x}) := k\mathbf{\Psi}_{\mathrm{V}}^{k}(\mathbf{e})(\mathbf{x}) + \frac{1}{k}\mathbf{grad}_x \Psi_V^k\left(div_\Gamma \mathbf{e}\right)(\mathbf{x}) \qquad \mathbf{x} \notin \Gamma,$$

$$\mathbf{\Psi}_{DL}^{k}(\mathbf{e})(\mathbf{x}) := \mathbf{curl}_x \mathbf{\Psi}_{\mathrm{V}}^{k}(\mathbf{e})(\mathbf{x}), \qquad \mathbf{x} \notin \Gamma.$$

Potentials = mappings of functions on $\Gamma$ to functions on $\Omega^+ \cup \Omega^-$

$$\Psi_V^k(\phi)(\mathbf{x}) := \int_\Gamma \phi(\mathbf{y}) G_k(\mathbf{x}-\mathbf{y})\, dS(\mathbf{y})$$

$$\mathbf{\Psi}_{\mathrm{V}}^{k}(\mathbf{u})(\mathbf{x}) := \int_\Gamma \mathbf{u}(\mathbf{y}) G_k(\mathbf{x}-\mathbf{y})\, dS(\mathbf{y})$$

$$G_k(\mathbf{x},\mathbf{y}) = \frac{e^{ik|\mathbf{x}-\mathbf{y}|}}{4\pi|\mathbf{x}-\mathbf{y}|}$$

# Boundary Integral Operators

We introduce the **boundary integral operators**:

$$\mathbf{S}_k := \{\gamma_t \mathbf{\Psi}_{SL}^k\}_\Gamma = \{\gamma_N \mathbf{\Psi}_{DL}^k\}_\Gamma \quad \mathbf{C}_k := \{\gamma_t \mathbf{\Psi}_{DL}^k\}_\Gamma = \{\gamma_N \mathbf{\Psi}_{SL}^k\}_\Gamma \quad \text{with } \{\bullet\}_\Gamma := \frac{1}{2}(\gamma^+ - \gamma^-)$$

$$\mathbf{S}_k, \mathbf{C}_k : \boldsymbol{H}_x^{-1/2}(div_\Gamma, \Gamma) \mapsto \boldsymbol{H}_x^{-1/2}(div_\Gamma, \Gamma)$$

They satisfy the **jump conditions**:

$$[\gamma_t \mathbf{\Psi}_{SL}^k]_\Gamma = [\gamma_N \mathbf{\Psi}_{DL}^k]_\Gamma = 0 \quad [\gamma_N \mathbf{\Psi}_{SL}^k]_\Gamma = [\gamma_t \mathbf{\Psi}_{DL}^k]_\Gamma = -Id \quad \text{with } [\bullet]_\Gamma := \gamma^+ - \gamma^-$$

Apply **traces** and **jump conditions** to (1):

$$\begin{aligned}
\gamma_{\mathbf{t}}^- \mathbf{e} &= \tfrac{1}{2}\gamma_{\mathbf{t}}^- \mathbf{e} + \mathbf{C}_\kappa(\gamma_{\mathbf{t}}^- \mathbf{e}) + \mathbf{S}_\kappa(\gamma_N^- \mathbf{e}) \,, \\
\gamma_{\mathbf{t}}^+ \mathbf{e} &= \tfrac{1}{2}\gamma_{\mathbf{t}}^+ \mathbf{e} - \mathbf{C}_\kappa(\gamma_{\mathbf{t}}^+ \mathbf{u}) - \mathbf{S}_\kappa(\gamma_N^+ \mathbf{e}) \,, \\
\gamma_N^- \mathbf{e} &= \mathbf{S}_\kappa(\gamma_{\mathbf{t}}^- \mathbf{e}) + \tfrac{1}{2}\gamma_N^- \mathbf{e} + \mathbf{C}_\kappa(\gamma_N^- \mathbf{e}), \\
\gamma_N^+ \mathbf{e} &= -\mathbf{S}_\kappa(\gamma_{\mathbf{t}}^+ \mathbf{e}) + \tfrac{1}{2}\gamma_N^+ \mathbf{e} - \mathbf{C}_\kappa(\gamma_N^+ \mathbf{e}).
\end{aligned} \qquad (2)$$

# Coupled Boundary Integral Equations

By means of the scaled traces:

$$\left(\boldsymbol{\zeta}^{\pm}, \boldsymbol{\lambda}^{\pm}\right) = \left(\gamma_t^{\pm}\mathbf{e}, \frac{k^{\pm}}{\mu^{\pm}}\gamma_N^{\pm}\mathbf{e}\right)$$

Transmission conditions become:

$$\boldsymbol{\zeta}^{-} = \boldsymbol{\zeta}^{+} + \gamma_t^{\pm}\mathbf{e}_i, \qquad \boldsymbol{\lambda}^{-} = \boldsymbol{\lambda}^{+} + \gamma_t^{\pm}\mathbf{h}_i$$

By means of the Calderon projectors:

$$P_k^{-} := \begin{pmatrix} 1/2\, Id + \mathbf{C}_k & \mathbf{S}_k \\ \mathbf{S}_k & 1/2\, Id + \mathbf{C}_k \end{pmatrix}, \; P_k^{+} := \begin{pmatrix} 1/2\, Id - \mathbf{C}_k & -\mathbf{S}_k \\ -\mathbf{S}_k & 1/2\, Id - \mathbf{C}_k \end{pmatrix}$$

… (2) becomes:

$$\begin{pmatrix} -\frac{1}{2}\,\mathrm{Id} + \mathbf{C}_{\kappa_-} & \frac{\mu_s}{\kappa_-}\mathbf{S}_{\kappa_-} \\ \frac{\kappa_-}{\mu_s}\mathbf{S}_{\kappa_-} & -\frac{1}{2}\,\mathrm{Id} + \mathbf{C}_{\kappa_-} \end{pmatrix} \begin{pmatrix} \boldsymbol{\zeta}^{-} \\ \boldsymbol{\lambda}^{-} \end{pmatrix} = 0\,,$$

$$\begin{pmatrix} -\frac{1}{2}\,\mathrm{Id} - \mathbf{C}_{\kappa_+} & -\frac{\mu_0}{\kappa_+}\mathbf{S}_{\kappa_+} \\ -\frac{\kappa_+}{\mu_0}\mathbf{S}_{\kappa_+} & -\frac{1}{2}\,\mathrm{Id} - \mathbf{C}_{\kappa_+} \end{pmatrix} \begin{pmatrix} \boldsymbol{\zeta}^{+} \\ \boldsymbol{\lambda}^{+} \end{pmatrix} = 0\,. \qquad (3)$$

# Mapping Operators on the Boundary

Different equivalent formulas for mapping operators can be obtained from (3) (e.g top equation $\Rightarrow$ *non-symmetric* expression)

**We look for a symmetric expression by means of BEM operators.**

$$\mathbf{T}^{-} := \left[ \frac{k^{-}}{\mu^{-}} \mathbf{S}_{k^{-}} - \left( \frac{1}{2} Id + \mathbf{C}_{k^{-}} \right) \left( \frac{\mu^{-}}{k^{-}} \mathbf{S}_{k^{-}} \right)^{-1} \left( -\frac{1}{2} Id + \mathbf{C}_{k^{-}} \right) \right]$$

$$\mathbf{T}^{+} := \left[ -\frac{k^{+}}{\mu_0} \mathbf{S}_{k^{+}} + \left( \frac{1}{2} Id - \mathbf{C}_{k^{+}} \right) \left( \frac{\mu_0}{k^{+}} \mathbf{S}_{k^{+}} \right)^{-1} \left( -\frac{1}{2} Id - \mathbf{C}_{k^{+}} \right) \right]$$

$$\mathbf{T}^{+}, \mathbf{T}^{-} \text{ continuous mappings}: \boldsymbol{H}_x^{-1/2}(div_\Gamma, \Gamma) \mapsto \boldsymbol{H}_x^{-1/2}(div_\Gamma, \Gamma)$$

After some computations:

$$\mathbf{T}^{-}\zeta = \frac{k^{-}}{\mu^{-}} \mathbf{S}_{k^{-}} \zeta + \left( \frac{1}{2} Id + \mathbf{C}_{k^{-}} \right) \boldsymbol{\lambda}^{-}$$

$$\mathbf{T}^{+}\zeta^{+} = \left( -\frac{k^{+}}{\mu_0} \mathbf{S}_{k^{+}} \right) \zeta^{+} + \left( \frac{1}{2} Id - \mathbf{C}_{k^{+}} \right) \boldsymbol{\lambda}^{+}$$

and

$$\left( -\frac{1}{2} Id + \mathbf{C}_{k^{-}} \right) \zeta + \frac{\mu^{-}}{k^{-}} \mathbf{S}_{k^{-}} \boldsymbol{\lambda}^{-} = 0$$

$$\left( \frac{1}{2} Id + \mathbf{C}_{k^{+}} \right) \left( \zeta - \gamma_t^{+} \mathbf{e}_i \right) + \frac{\mu_0}{k^{+}} \mathbf{S}_{k^{+}} \boldsymbol{\lambda}^{+} = 0$$

where $\zeta^{-} = \zeta$

# Mapping Operators on the Boundary II

Recalling the transmission condition:

$$\mathbf{T}^{-}\gamma_t^{-}\mathbf{e} - \mathbf{T}^{+}\gamma_t^{+}\mathbf{e} = \frac{k^{+}}{\mu_0}\gamma_N^{+}\mathbf{e} = \gamma_t^{+}\mathbf{h}_i$$

$$\begin{pmatrix} \frac{k^-}{\mu^-}\mathbf{S}_{k^-} + \frac{k^+}{\mu_0}\mathbf{S}_{k^+} & \left(\frac{1}{2}Id + \mathbf{C}_{k^-}\right) & \left(-\frac{1}{2}Id + \mathbf{C}_{k^+}\right) \\ \left(-\frac{1}{2}Id + \mathbf{C}_{k^-}\right) & \frac{\mu^-}{k^-}\mathbf{S}_{k^-} & 0 \\ \left(\frac{1}{2}Id + \mathbf{C}_{k^+}\right) & 0 & \frac{\mu_0}{k^+}\mathbf{S}_{k^+} \end{pmatrix} \cdot \begin{pmatrix} \zeta \\ \boldsymbol{\lambda}^- \\ \boldsymbol{\lambda}^+ \end{pmatrix} = \begin{pmatrix} \gamma_t^+\mathbf{h}_i + \frac{k^+}{\mu_0}\mathbf{S}_{k^+}\left(\gamma_t^+\mathbf{e}_i\right) \\ 0 \\ \left(\frac{1}{2}Id + \mathbf{C}_{k^+}\right)\left(\gamma_t^+\mathbf{e}_i\right) \end{pmatrix} \quad (4)$$

***Note***

First equation – is the transmission condition – effective only on $\Gamma_a$.
Second and third equations in (4) involve relations between integral operators defined on the whole of $\Gamma$.

## Remark on "spurious resonances"

- If $k_+^2$ corresponds to a Dirichlet Maxwell eigenvalue of $\Omega_-$, the variational problem fails to possess a unique solution.
- Although solution of (4) may no longer be unique fields obtained from (1) remain unique.
- Nevertheless this situation causes numerical instabilities. In our case: 50% increase in number of iterations.

# Galerkin BEM Formulation

We project (4) as follows:

- First equation is tested with $\mu \in \mathbf{H}_x^{-1/2}(\mathrm{div}_\Gamma, \Gamma_a)$
- Second and third equations are tested with $\xi, \nu \in \mathbf{H}_x^{-1/2}(\mathrm{div}_\Gamma, \Gamma)$

$\Gamma$ will be approximated by a triangulation $\Gamma_h$ composed of flat triangles. We assume that the boundary of $\Gamma_a$ is approximately resolved by edges of $\Gamma_h$

Construct finite dimensional subspaces:

$\mathbf{v}_h \subset \mathbf{H}_x^{-1/2}(\mathrm{div}_\Gamma, \Gamma_a)$ $\mathbf{vv}_h \subset \mathbf{H}_x^{-1/2}(\mathrm{div}_\Gamma, \Gamma)$ edge elements,

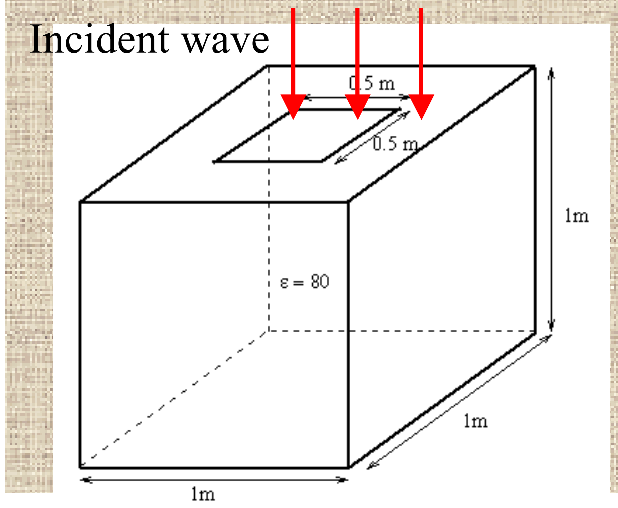
Test and shape functions = **RWG vectorial functions**

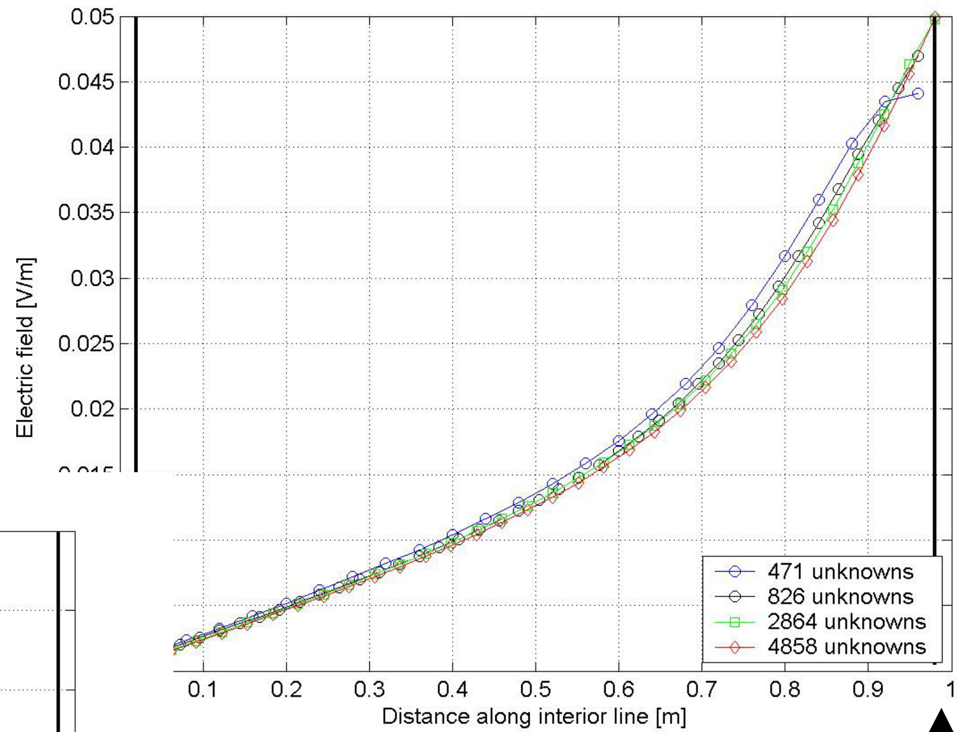
This will give a space of piecewise linear vectorfields on $\Gamma$ whose ``surface normal components" are continuous across edges of triangles.

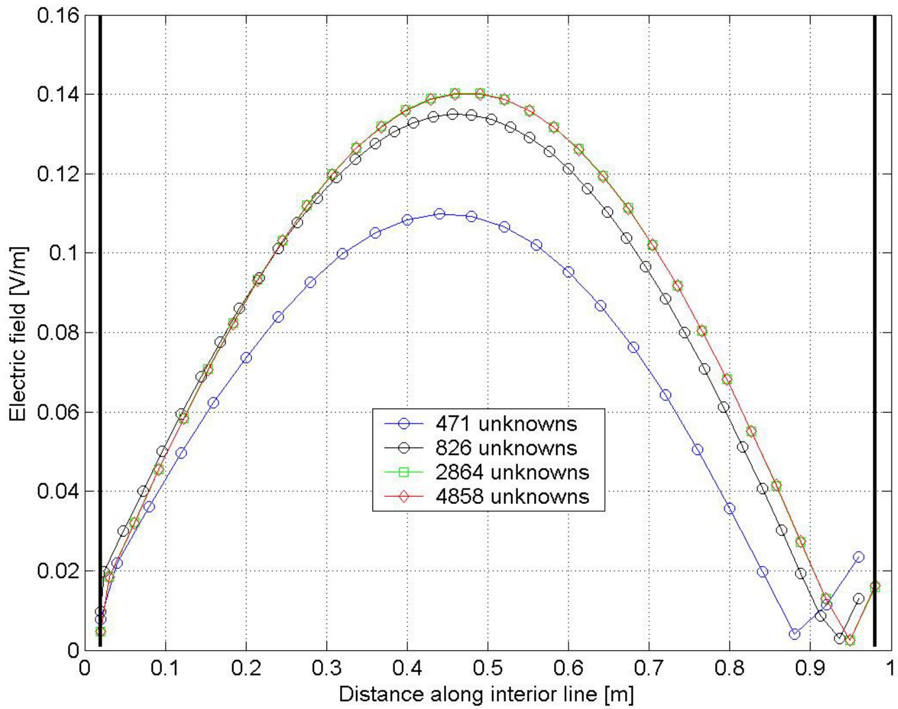
Square system of **$2N+N_A$**:

- N – number of edges in a triangulation $\Gamma_h$ of $\Gamma$
- $N_A$ – number of edges **only within the aperture** $\zeta = 0$ on $\partial\Gamma_a$

# Metallic Container Filled With Sea Water




Aperture Position

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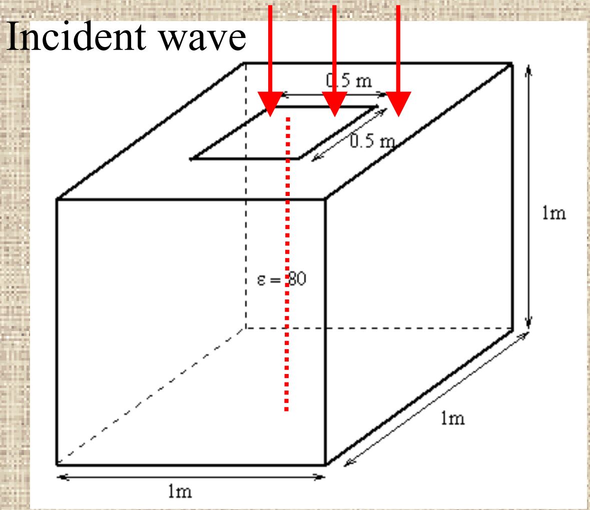
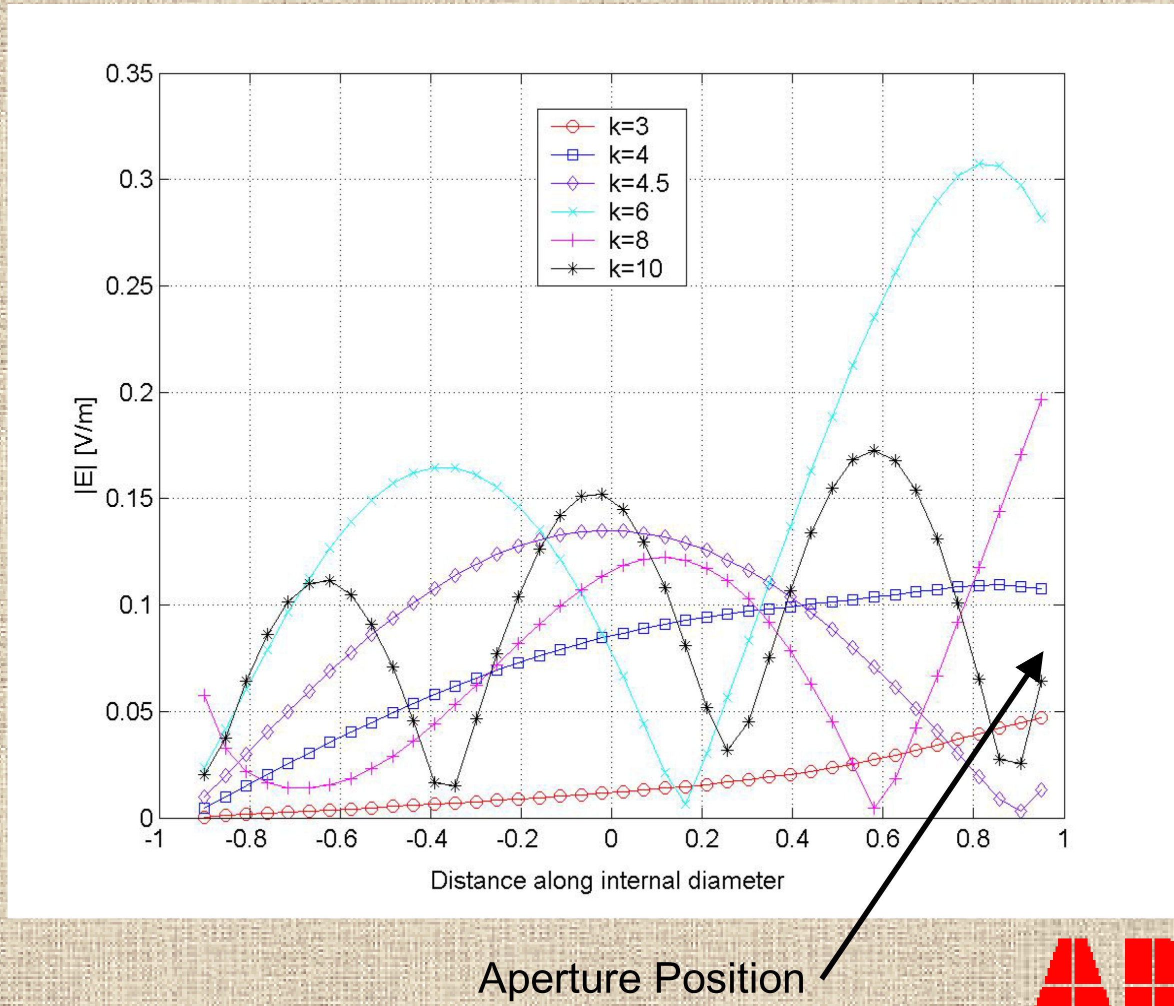
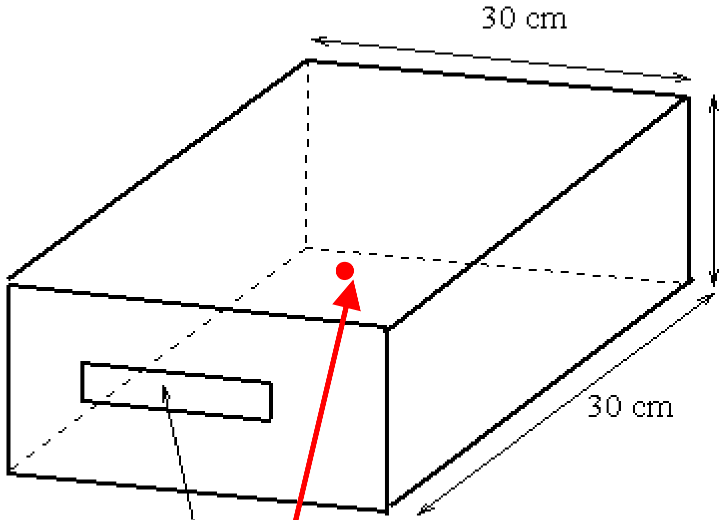
# Metallic Container Filled With Sea Water II



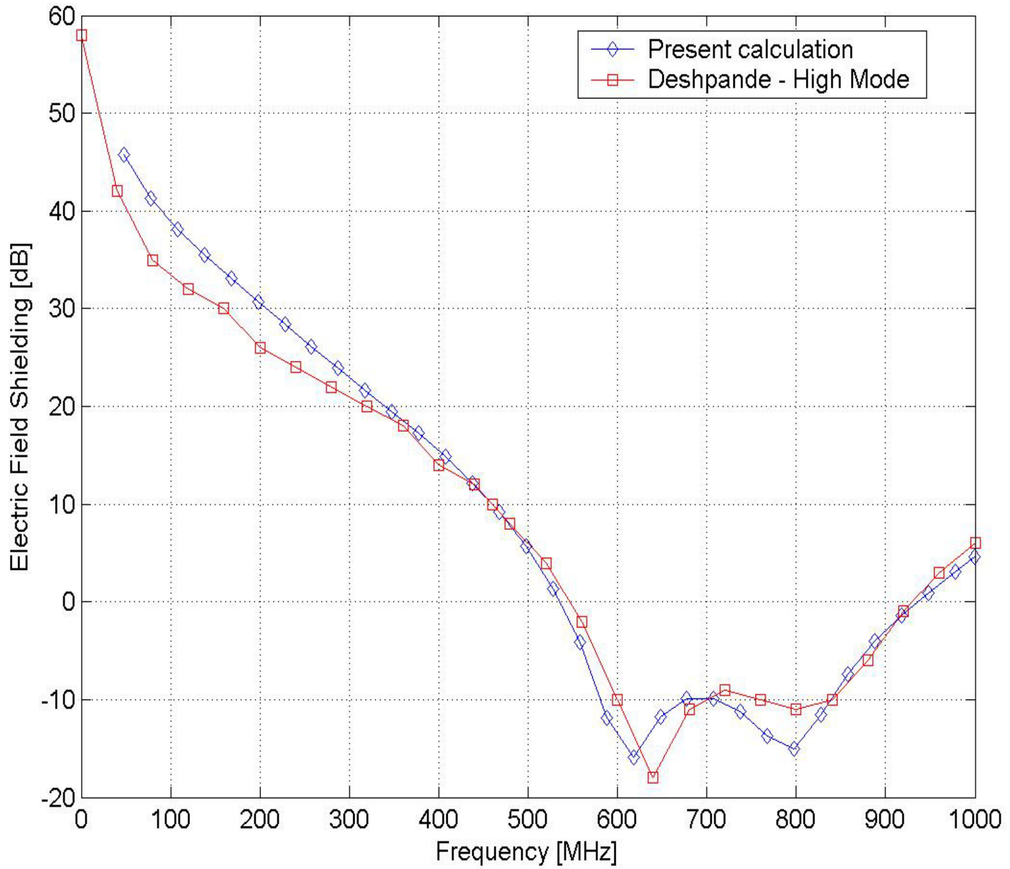

Electromagnetic field inside the container – see red line on the geometry plot



# Metallic Casing with 1 Aperture




Shielding efficiency measured in the centre of the enclosure (15, 6, 15) cm.




EFS factor:

$$EFS(x) = -20\log\left|\frac{\mathbf{E}}{\mathbf{E_i}}\right| \; [dB]$$



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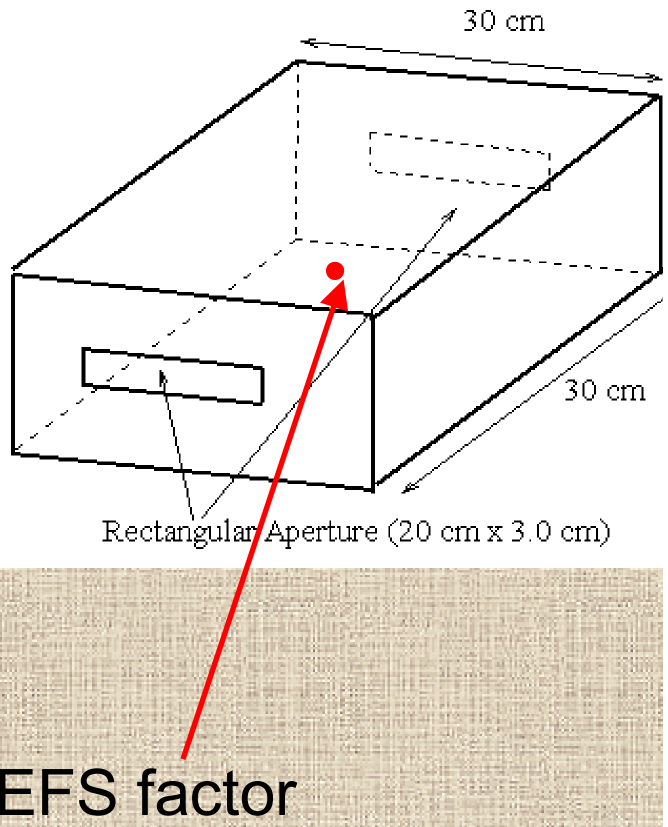
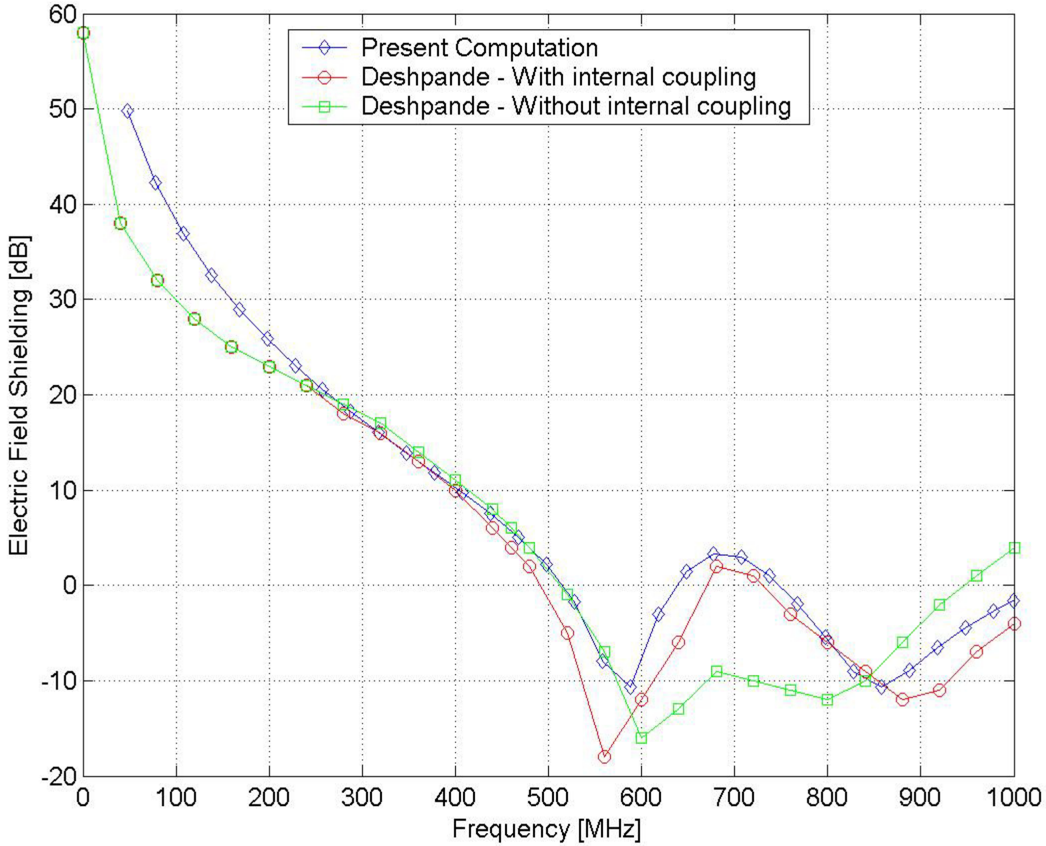
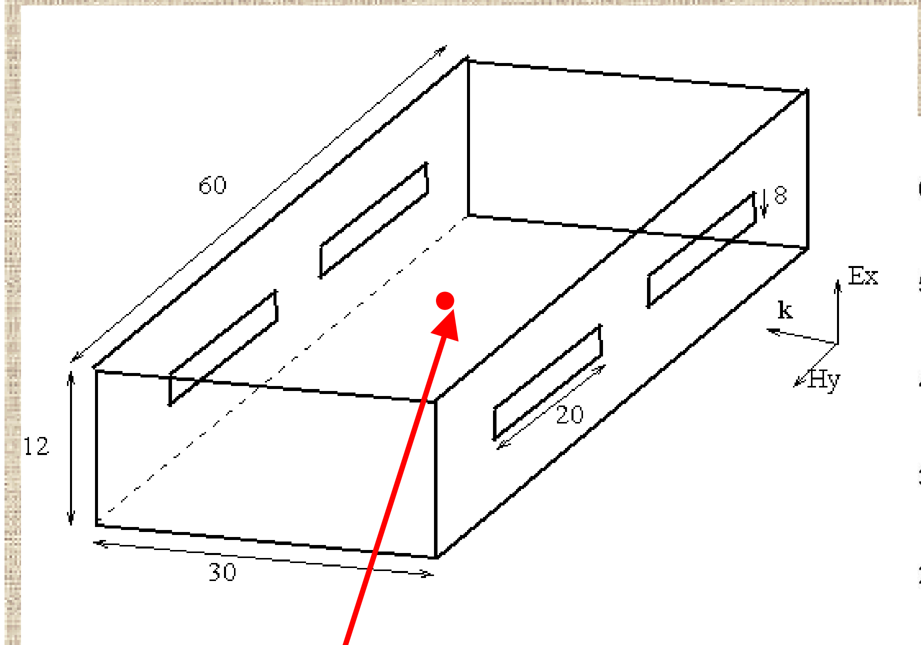
# Metallic Casing with 2 Apertures




Shielding efficiency measured in the centre of the enclosure (15, 6, 15) cm.

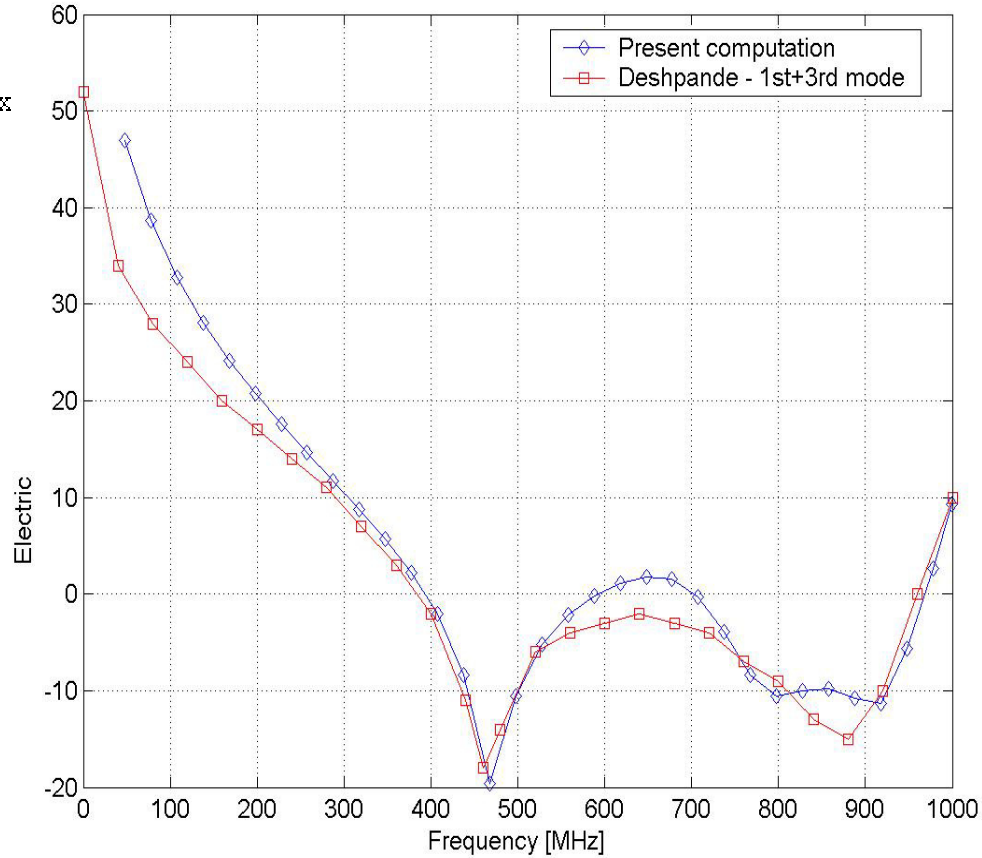


# Metallic Casing with 4 Apertures




EFS factor

Shielding efficiency measured in the centre of the enclosure (30, 6, 15) cm.



# Conclusions

- Direct Boundary Integral Equation Approach: use of physical unknowns.
- Use Electric-to-Magnetic Mapping – accommodate with no problem both PEC and Transmission BC.
- Obtains symmetrical formulation.
- Able to treat configurations independent of the geometry.
- Able to treat $\varepsilon_{int} \neq \varepsilon_{ext}$ and/or $\mu_{\text{int}} \neq \mu_{ext}$