Technische Universität Graz

13. Workshop on

Fast Boundary Element Methods in Industrial Applications

Söllerhaus, 23.–26.10.2015

U. Langer, M. Schanz, O. Steinbach, W. L. Wendland (eds.)

Berichte aus dem Institut für Numerische Mathematik

Book of Abstracts 2015/4

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Program

Friday, October 23, 2015	
16.00	Coffee
17.00–17.45	C. Erath (Darmstadt, Germany) A non symmetric FVM–BEM coupling method
17.45–18.30	J. Dölz (Basel, Switzerland) An interpolation–based fast multipole method for higher order boundary elements on parametric surfaces
18.30	Dinner
Saturday, October 24, 2015	
9.00–9.45	L. Desiderio (Paris, France) An $\mathcal{H}$–matrix based direct solver for boundary element method in 3D elastodynamics
9.45–10.30	M. Merta (Ostrava, Czech Republic) Acceleration of the BEM4I library using the Intel Xeon Phi coprocessors
10.30–11.00	Coffee
11.00–11.45	G. Unger (Graz, Austria) Numerical analysis of boundary element methods for Maxwell's eigenvalue problem
11.45-12.30	C. Urzua–Torres (Zürich, Switzerland) Operator preconditioning for the hypersingular operator over 3D quasi–flat screens
12.30	Lunch
15.00–15.45	M. Zank (Graz, Austria) Time–domain boundary integral equations for the wave equation
15.45–16.30	E. Spindler (Zürich, Switzerland) Well–conditioned single trace BEM for electromagnetic scattering at composite objects
17.00–17.45	R. Hiptmair (Zürich, Switzerland) Cut–free BEM for scalar magnetic potentials
17.45–18.30	H. Harbrecht (Basel, Switzerland) Efficient approximation of random fields for numerical applications
18.30	Dinner

Sunday, October 25, 2015	
9.00–9.45	W. L. Wendland (Stuttgart, Germany) The Navier problem for the Darcy Forchheimer Brinkmann system in Lipschitz domains
9.45–10.30	R. Grzhibovskis (Saarbrücken, Germany) Boundary–domain integral formulation for boundary value problem involving the Laplace–Beltrami operator
10.30–11.00	Coffee
11.00–11.45	U. Langer (Linz, Austria) Space–time isogeometric analysis of parabolic evolution problems
11.45–12.30	O. Steinbach (Graz, Austria) Space–time finite and boundary element methods for parabolic problems
12.30	Lunch
13.30–18.00	Hiking Tour
18.30	Dinner
Monday, October 26, 2015	
8.00–9.00	Breakfast

An $\mathcal{H}$-matrix based direct solver for boundary element method in 3D elastodynamics

S. Chaillat, P. Ciarlet Jr., L. Desiderio
ENSTA Paristech, France

The boundary element method is well suited to treat seismic wave propagation problems in semi-infinite regions. Although the resulting matrices are dense, it is possible to find evaluation procedures that are fast and avoid their explicit storage, in order to compress the informations and give a data-sparse representation. In the present work, we propose a fast method to solve the boundary element method system in 3D frequencydomain elastodynamics. Using the $\mathcal{H}$-matrix arithmetic and low-rank approximations (performed with Adaptive Cross Approximation), we derive a fast direct solver. We assess the numerical efficiency and accuracy on the basis of numerical results obtained for problems having known solutions. In particular, we study the efficiency of low-rank approximations when the frequency is increased.

An interpolation-based fast multipole method for higher order boundary elements on parametric surfaces

J. Dölz, H. Harbrecht, M. Peters
Universität Basel, Switzerland

We propose a black-box higher order fast multipole method for solving boundary integral equations on parametric surfaces in three dimensions. Such piecewise smooth surfaces are the topic of recent studies in isogeometric analysis. Due to the exact surface representation, the rate of convergence of higher order methods is not limited by approximation errors of the surface. An element-wise clustering yields a balanced cluster tree and an efficient numerical integration scheme for the underlying Galerkin method. By performing the interpolation for the fast multipole method directly on the reference domain, we reduce the cost complexity in the polynomial degree by one order. This gain is independent of the application of either $\mathcal{H}$- or $\mathcal{H}^2$-matrices. In fact, we point out several simplifications in the construction of $\mathcal{H}^2$-matrices, which are a by-product of the surface representation.

A non symmetric FVM-BEM coupling method

C. Erath[1], G. Of[2], F.–J. Sayas[3]
[1]TU Darmstadt, Germany, [2]TU Graz, Austria, [3]University of Delaware, USA

Recently, the coupling of the finite volume method (FVM) and the boundary element method (BEM) was shown to be an efficient method to solve transmission problems, in particular for problems of the fluid dynamics. In this talk we focus on the new non-symmetric FVM-BEM coupling approach, which results in a smaller system of linear equations than the previous three field FVM-BEM couplings. We consider the prototype model for flow and transport of a concentration in porous media in an interior domain and couple it with a diffusion process in the corresponding unbounded exterior domain. This coupling idea might also have another interpretation: instead of (the missing) Dirichlet and/or Neumann boundary conditions of the interior transport we assume a diffusion process in the corresponding (unbounded) exterior domain, which "replaces" the boundary values. The FVM-BEM discretisation of this model problem provides naturally conservation of local fluxes and with an upwind option also stability in the convection dominated case. Note that the coupling with BEM avoids the truncation of the unbounded exterior domain. We aim to provide a first rigorous analysis of the discrete system for different model parameters; stability, convergence, and a priori estimates. This includes the use of an implicit stabilization, known from the finite element and boundary element method coupling, but here extended to the different interior problem. The stabilization is only needed for theoretcial purposes. Numerical examples illustrate the strength of the chosen method. Some comparisons with the three field FVM-BEM coupling conclude the presentation.

References

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[3] C. Erath: A new conservative numerical scheme for flow problems on unstructured grids and unbounded domains. J. Comput. Phys. 245 (2013) 476–492.

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Boundary-domain integral formulation for boundary value problem involving the Laplace-Beltrami operator

R. Grzhibovskis

Saarland University, Saarbrücken, Germany

A boundary value problem for the Laplace-Beltrami operator on a smooth two-dimensional surface embedded in $\mathbb{R}^3$ is considered. As in the case of an inhomogeneous heat transfer [1], a suitable parametrix (Levi function) is found and an integral formulation of the problem is derived. This formulation involves geometrical properties of the surface. Furthermore, besides the usual boundary integrals the integration along the surface is present.

A numerical method of finding the approximate solution is derived similarly to the corresponding case in $\mathbb{R}^3$ [2]. Several key differences and similarities to the popular finite element methods are discussed. Some aspects of implementation are commented on and several numerical examples are presented.

References

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Efficient approximation of random fields for numerical applications

H. Harbrecht, M. Peters, M. Siebenmorgen
Universität Basel, Switzerland

This talk is dedicated to the rapid computation of separable expansions for the approximation of random fields. We consider approaches based on techniques from the approximation of non-local operators on the one hand and based on the pivoted Cholesky decomposition on the other hand. Especially, we provide an a-posteriori error estimate for the pivoted Cholesky decomposition in terms of the trace. Numerical examples are provided to validate and quantify the presented methods.

Cut–free BEM for scalar magnetic potentials

R. Hiptmair, O. Stein
ETH Zürich, Switzerland

We consider a standard linear eddy current problem in frequency domain. The magnetic offset field $\mathbf{H}$ outside the conductors satisfies $\operatorname{curl}\mathbf{H} = \mathbf{0}$. In an $\mathbf{H}$-based model this algebraic constraint can be taken into account by means of a scalar magnetic potential Ψ satisfying $\mathbf{H} = \operatorname{grad}\Psi$, provided that the first Betti number of the conducting region vanishes. Further, Ψ will be a harmonic function.
However, if the conductor has "holes", for instance, if it shaped like a torus, then we must introduce so-called Seiffert surfaces or "cuts", which close the holes, and allow constant jumps of magnetic scalar potentials across them. Cuts also enter the boundary integral representation formulas for Ψ. Those reduce to only a few terms thanks to cancellation of contributions from the two sides of a cut. It turns out that, eventually, cuts contribute only a few scalar valued functionals to boundary integral equations for traces of magnetic potentials.
On the one hand, these functionals depend only on the boundary of the cuts. On the other hand their evaluation seems to entail integration over cuts, which would require meshing them in the context of a boundary element method. We demonstrate, how to reduce integration to the boundaries of the cuts, thus avoiding finding and meshing them. The resulting algorithm still involves the computation of solid angles. This raises issues of numerical stability.

Space–time isogeometric analysis of parabolic evolution problems

U. Langer, S. E. Moore, M. Neumüller
Johannes Kepler Universität Linz, Austria

We present and analyze a new stable space-time Isogeometric Analysis (IgA) method for the numerical solution of parabolic evolution equations in fixed and moving spatial computational domains. The discrete bilinear form is elliptic on the IgA space with respect to a discrete energy norm. This property together with a corresponding boundedness property, consistency and approximation results for the IgA spaces yields an a priori discretization error estimate with respect to the discrete norm. The theoretical results are confirmed by several numerical experiments with low- and high-order IgA spaces.

Acceleration of the BEM4I library using the Intel Xeon Phi coprocessors

M. Merta, J. Zapletal
VSB TU Ostrava, Czech Republic

The Intel Xeon Phi coprocessors provide an efficient tool for the acceleration of scientific codes. Although a naive implementation is easily achieved using Intel's offload pragmas, to fully exploit the many-core architecture it is usually necessary to optimize the computationally most intensive kernels for aligned memory access, loop vectorization, etc. The BEM4I library currently accelerates the assembly of the system matrices for the Laplace and Lamé operators in 3D. We present the results of the numerical experiments carried out on the Salomon supercomputer installed at the IT4Innovations National Supercomputing Center in August 2015. With its 864 Xeon Phi 7120P cards it is currently the biggest installation of the Intel's coprocessors in Europe. The experiments include the assembly of full system matrices and preliminary results for the ACA sparsification. In connection with the Espresso domain decomposition library we provide additional numerical benchmarks for the boundary element tearing and interconnecting method.

Well–conditioned single trace BEM for electromagnetic scattering at composite objects

X. Claeys[1], R. Hiptmair[2], E. Spindler[2]

[1]Laboratoire J.–L. Lions, Paris 6, France, [2]ETH Zürich, Switzerland

We consider time harmonic electromagnetic scattering at an object that has piecewise constant permittivity and assume that all homogeneous parts of the scatterer are curvilinear polygonal Lipschitz. We focus on the corresponding electric field integral equation. Since we have to cope with unbounded domains, boundary element approaches are a convenient tool for numerical treatment. The widely used classical first-kind approach (PMCHWT) is ill-conditioned with no suitable preconditioner available.

We establish a well-conditioned second-kind boundary element approach which uses $H^{\delta}_{\mathbf{t}}$ as test space for a fixed δ, $0 < \delta < \frac{1}{2}$ depending on the regularity of the Maxwell solution. Here, $H^{\delta}_{\mathbf{t}}$ is the space of tangential vector fields of Sobolev regularity H^{δ}. As trial space we use its dual space $H^{-\delta}_{\mathbf{t}}$, $L_{\mathbf{t}}$ playing the role of the pivot space. This setting allows us to use discontinuous vector fields to approximate the unknown boundary data in a Galerkin discretisation. We take the boundary element space of piecewise constant tangential vector fields on each element of the mesh to discretise the new formulation.

Numerical results show competitive accuracy of the new second-kind approach, bounded condition numbers of the Galerkin matrices with respect to the mesh size and superior convergence of GMRES.

References

[1] X. Claeys, R. Hiptmair, E. Spindler: A second-kind Galerkin boundary element method for scattering at composite objects. BIT 55 (2015) 33–57.

[2] X. Claeys, R. Hiptmair, E. Spindler: Second-kind boundary integral equations for scattering at composite partly impenetrable objects. Technical Report No. 19, ETH Zürich, 2015.

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Space–time finite and boundary element methods for parabolic problems

O. Steinbach
TU Graz, Austria

In most cases, finite and boundary element methods for time–dependent partial differential equations rely on time–stepping schemes. Although such an approach allows for a subsequent solution of the discrete system, it may not reflect the behavior of the solution properly, at least from an approximation point of view. For the model problem of the heat equation we will consider finite and boundary element methods with respect to general decompositions of the space–time domain and its boundary into finite and boundary elements, respectively. In particular, such an approach allows for an adaptive refinement simultaneously in space and time. Moreover, the global solution of the overall space–time system can be done in parallel, in contrast to more standard time discretization schemes. Here we will present a stability and error analysis of space–time finite and boundary element methods, and we present some numerical results which indicate the potential of the proposed approach.

Numerical analysis of boundary element methods for Maxwell's eigenvalue problem

G. Unger
] TU Graz, Austria

We consider a Galerkin approximation of boundary integral formulations of Maxwell's eigenvalue problem in bounded and unbounded domains. An analysis of the boundary integral formulations and their numerical approximations is given in the framework of eigenvalue problems for holomorphic Fredholm operator-valued functions. We show that the Galerkin approximations yield a so-called regular convergent sequence to the underlying operator-valued function of the eigenvalue problem. This allows us to apply the general results of the numerical analysis of [2,3] which guarantee convergence of the eigenvalues as well as of the eigenspaces and which provide error estimates. The numerical examples, presented in [1], confirm the theoretical results.

References

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Operator preconditioning for the hypersingular operator over 3D quasi–flat screens

R. Hiptmair[1], C. Urzúa–Torres[1], C. Jerez–Hanckes[2]
[1]ETH Zürich, Switzerland
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In this presentation we propose a new Calderón preconditioner for the hypersingular operator on 3D screens arising from the Laplacian. For its construction, we use operator preconditioning [2] and the bilinear form induced by its inverse boundary integral operator (BIO) over the disk, which can be obtained from Fabrikant's work [1]. As predicted by our predecessors, the inverse BIO is a (modified) weakly singular operator that incorporates the distance to the boundary $\partial\Gamma$ of the screen Γ, in an analogous way to the function $M(x, y)$ in the 2D case [3,4].
Our numerical results show the optimality of our preconditioner when applied to different flat screens and an almost optimal behaviour for quasi-flat screens. Furthermore, this preconditioning technique can be used on non-uniform meshes. This property poses a great advantage, as solutions of boundary integral equations on screens feature a square-root type singularity at $\partial\Gamma$, which can be resolved by refining the mesh towards the boundary.

References

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The Navier problem for the Darcy Forchheimer Brinkman system in Lipschitz domains

W. L. Wendland
Universität Stuttgart, Germany

The lecture ist based on the paper [1], joint work with M. Lanza de Cristoforis and M. Kohr. We study the Navier boundary value problem and the nonlinear Darcy–Forchheimer–Brinkman system as a compact perturbation of the linear Stokes system. The corresponding boundary integral equations turn out to be Fredholm mappings of index zero, and related a priori estimates allow an analysis of the nonlinear equations. Then Schauder's fixed point theorem provides the solvability of the nonlinear equations providing the solution of the Darcy–Forchheimer–Brinkman model for viscous flows.

References

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Time–domain boundary integral equations for the wave equation

M. Zank
TU Graz, Austria

In this talk we consider the time–domain boundary integral equations associated to the wave equation.
The first aim is to derive the so-called Kirchhoff's formula which is a representation formula for the wave equation's solution by boundary integral operators. To obtain Kirchhoff's formula in the distributional sense we apply the Laplace transform to the wave equation, analyse the resulting time harmonic problem in the Laplace domain and transform back to the time dependent problem. For that reason, the Laplace transform is considered for causal tempered distributions. Further, the time harmonic problem is analysed explicitly in terms of the wavenumber. A last ingredient is a kind of Paley-Wiener theorem for transforming back to the time dependent problem.
As a second aim we examine the coercivity of boundary integral operators in the time–domain. Therefore, some anisotropic Sobolev spaces are introduced and the loss of regularity in time is compared for different approaches.

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2014/3 M. Neumüller, O. Steinbach: An energy space finite element approach for distributed control problems.

2014/4 L. John, O. Steinbach: Schur complement preconditioners for boundary control problems.

2014/5 O. Steinbach: Partielle Differentialgleichungen und Numerik.

2014/6 T. Apel, O. Steinbach, M. Winkler: Error estimates for Neumann boundary control problems with energy regularization.

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