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Program

Thursday, October 12, 2017	
15.00	Coffee
16.00–16.45	L. Banz (Salzburg) Improved stabilization technique for frictional contact problems solved with hp BEM
16.45–17.30	H. Gimperlein (Edinburgh, Paderborn) Adaptive finite elements for the parabolic fractional obstacle problem
17.30–17.45	Break
17.45–18.30	X. Claeys (Paris) Second kind boundary integral equation for multi–subdomain diffusion problems
18.30	Dinner
Friday, October 13, 2017	
8.00–9.00	Breakfast
9.00–9.45	H. Gimperlein (for E. P. Stephan, Hannover) Adaptive and higher–order time domain boundary elements for the wave equation
9.45–10.30	D. Pölz (Graz) Time–domain boundary elements for elastodynamic trusses
10.30–11.00	Coffee
11.00–11.45	M. Scroggs (London) Solving integral equations for electromagnetic scattering using Bempp
11.45–12.30	D. Lukas (Ostrava) Coupling of finite and boundary elements for transient eddy current problems
12.30	Lunch
14.00–14.45	A. Kleanthous (London) Calderon preconditioning for electromagnetic scattering of dielectric objects
14.45–15.30	C. Urzua–Torres (Zürich) Operator preconditioning for the electric field integral equation on the disk
15.30–16.00	Coffee
16.00–16.45	C. Erath (Darmstadt) A non–symmetric FEM–BEM coupling method for a parabolic–elliptic interface problem
16.45–17.30	G. Of (Graz) On the non–symetric BEM–FEM coupling for the Stokes problem
18.30	Dinner

Saturday, October 14, 2017	
8.00–9.00	Breakfast
9.00–9.45	S. Kurz (Darmstadt) Three–dimensional isogeometric BEM: The journey so far
9.45–10.30	J. Zapletal (Ostrava) Efficient analytic evaluation of boundary integral operators
10.30–11.00	Coffee
11.00–11.45	M. Bauer (Bayreuth) Additional adaptive methods for the adaptive cross approximation of BEM matrices
11.45–12.30	S. Dohr (Graz) Space–time BEM for the heat equation
12.30	Lunch
13.30–18.00	Hiking Tour
18.30	Dinner
Sunday, October 15, 2017	
8.00–9.00	Breakfast
9.00–9.45	O. Steinbach (Graz) Coercive space–time finite and boundary element methods
9.45–10.30	M. Zank (Graz) Space–time methods for the wave equation
10.30–11.00	Coffee

Improved stabilization technique for frictional contact problems solved with *hp*-BEM

Lothar Banz[1] Gregor Milicic[1], Nina Ovcharova[2]

[1]Universität Salzburg, Austria, [2]Universität der Bundeswehr, München, Germany

We improve the residual based stabilization technique for Signorini contact problems with Tresca friction in linear elasticity solved with *hp*-mixed BEM which has been recently analyzed by Banz et al. in Numer. Math. 135 (2017) pp. 217–263. The stabilization allows us to circumvent the discrete inf-sup condition and thus the primal and dual sets can be discretized independently. Compared to the above mentioned paper we are able to remove the dependency of the scaling parameter on the unknown Sobolev regularity of the exact solution and can thus also improve the convergence rate in the a priori error estimate. The second improvement is a rigorous a priori and a posteriori error analysis when the boundary integral operators in the stabilization terms, which are of the type $\langle Wu_{hp}, Wv_{hp}\rangle$, are approximated. The latter is of fundamental importance to keep the computational time small. We present numerical results in two and three dimensions to underline our theoretical findings, show the superiority of the *hp*-adaptive stabilized mixed scheme and the effect induced by approximating the stabilization term. Moreover, we show the applicability of the proposed method to the Coulomb frictional case for which we extend the a posteriori error analysis.

Additional adaptive methods for the adaptive cross approximation of BEM-Matrices

M. Bauer, M. Bebendorf
Universität Bayreuth, Germany

In the context of BEM, the adaptive cross approximation with its linear logarithmic complexity has turned out to be a fast method for the approximative solution of the appearing systems of linear equations. The algorithm uses a partition of the discrete operator with respect to the underlying geometry and approximates uniformly each block of the hierarchical structure regardless of the right hand side. In most cases the approximation of one block has not the same influence on the solution as one other. So, the universal approximation of the discrete operator is bought expensively in these cases with the generation of redundant information.
The aim of the talk is the connection of the ACA to the right hand side in order to decrease the memory requirement of the approximated matrix. In addition to this memory reduction we consider, whether an acceleration of the matrix approximation can be reached. Therefore we are going to use strategies from the adaptive BEM and introduce an error estimator for the ACA, which steers the approximation to the blocks with the highest precision gain. Furthermore a convergence result of the new ACA can be formulated with the help of the estimator convergence. Numerical examples show the functionality of the resulting algorithm.

Second kind boundary integral equation for multi-subdomain diffusion problems

Xavier Claeys[1,2], Ralf Hiptmair[3], Elke Spindler[3]
[1]LJLL UPMC, [2]INRIA Alpines, [3]ETH Zürich

We study elliptic boundary value problems where coefficients are piecewise constant with respect to a partition of space into Lipschitz subdomains, focusing on the case of jumping coefficients arising in the principal part of the partial differential operator. We propose a boundary integral equation of the second kind posed on the interfaces of the partition, and involving only one unknown trace function at each interface. We provide a detailed analysis of the corresponding integral operator, proving well- posedness. We also present numerical results that exhibit a systematically stable condition number for the associated Galerkin matrices, so that GMRES seems to enjoy fast convergence independent of the mesh resolution.

Space-time BEM for the heat equation

Stefan Dohr, Olaf Steinbach
TU Graz, Austria

The standard approach in space-time boundary element methods for discretizing variational formulations of boundary integral equations is using space-time tensor product spaces originating from a separate decomposition of the boundary Γ and the time interval $(0, T)$. However, this approach does not allow adaptive refinement in space and time simultaneously. This motivates the use of an arbitrary decomposition of the whole space-time boundary $\Sigma = \Gamma \times (0, T)$ into boundary elements. In this talk we consider the heat equation as a model problem and compare these two discretization methods. Moreover, when using space-time tensor product spaces we can construct a preconditioner for the first boundary integral equation by using the discretization of the hypersingular operator with respect to an appropriate dual mesh. The theoretical results are confirmed by numerical tests.

A non–symmetric FEM–BEM coupling method for a parabolic-elliptic interface problem

H. Egger, C. Erath, R. Schorr
TU Darmstadt, Germany

Based on the work of Johnson and Nédélec [2], MacCamy and Suri [1] proposed a non-symmetric coupling method to approximate the solution of a parabolic-elliptic interface problem. Such problems arise, e.g., in the modeling of eddy currents in the magneto quasi-static regime. Their analysis, however, relies on some regularity assumptions on the boundary Γ to ensure the compactness of the double layer operator. Furthermore, they only provide the quasi-optimality of a Galerkin approximation, i.e., an analysis of an additional time discretization was omitted. Therefore, Costabel, Ervin, and Stephan [3] consider a symmetric coupling version to overcome the difficulties of the non-symmetric coupling. More precisely, their analysis covers non-smooth domains and establishes error estimates also for a fully discrete scheme under some regularity assumptions. However, a direct application of the analysis in [3] for the non-symmetric coupling is not possible due to the lack of adjoint consistency. In the presented work, soon available as preprint [4], we extend the results of [3,4] to the non-symmetric coupling method on non-smooth domains, i.e., we prove well-posedness and provide quasi-optimal error estimates for Galerkin approximations. Furthermore, we present the first complete analysis for a time discretization of the semi-discrete non-symmetric coupling without using a (not available) duality argument. To avoid additional regularity assumptions on the data, we introduce a variant of the implicit Euler method. All results are established with respect to the natural energy norm under minimal smoothness assumptions. Several numerical examples verify our theoretical findings.

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Adaptive finite elements for the parabolic fractional obstacle problem

Heiko Gimperlein, Jakub Stocek
Heriot-Watt University, Edinburg, UK, University of Paderborn, Germany

The parabolic obstacle problem for the fractional Laplacian naturally arises in finance and biology for random walks driven by a pure jump Lévy process. We consider its finite element discretization, both as a variational inequality and in a mixed formulation. The a priori and a posteriori error analysis is discussed in both cases. We present both residual and hierarchical a posteriori error estimates, as well as the resulting adaptive mesh refinement procedures. The theory is illustrated by numerical examples.

Calderon preconditioning for electromagnetic scattering of dielectric objects

Antigoni Kleanthous, Timo Betcke, David Hewett
University College London, UK

In recent years Calderon preconditioning and appropriate use of basis functions has become a popular strategy to speed up the iterative solution of electromagnetic scattering problems. In this talk we discuss the application of Calderon preconditioners to dielectric scattering problems. We will derive the formulation, discuss its properties, and demonstrate its implementation in the boundary element library BEM++. An application of particular interest to us is the investigation of light scattering properties of ice crystals. We will briefly introduce this problem and demonstrate examples of applying Calderon preconditioners to ice crystal scattering problems.

Three–dimensional isogeometric BEM: The journey so far

J. Dölz[1], H. Harbrecht[1], S. Kurz[2], S. Schöps[2], F. Wolf[2]
[1]Universität Basel, Switzerland, [2]TU Darmstadt, Germany

One of the major drawbacks of isogeometric methods, namely the volume parametrization problem, can be avoided by the utilization of (indirect) boundary element methods, as explained in [4]. First analytic results towards the final goal of an isogeometric boundary element method for the Maxwell eigenvalue problem have been established. Via application of trace operators one can utilize the framework of [6] to generate a meaningful discretization of the two-dimensional Hilbert complex arising in the formulation of boundary element problems. Approximation estimates of the resulting spline spaces can be provided by generalizing the construction of quasi interpolants as in [2] for projectors which interpolate boundary values, heeding changes in the stability of the projectors. The resulting interpolation operators commute w.r.t. the surface differential operators, and make it possi-ble to utilize established techniques as in [1] to show adequateness of the isogeometric discretization.
Regarding implementation, the interpolation-based multipole method from [3] has been utilized to apply the isogeo-metric methodology to the Laplace and Helmholtz problem, for which numerical results are available [4]. The results are promising, but also show challenges of the higher order approach, that seem natural for methods utilizing para-metric mappings with tensor product structure. Moreover, first numerical results for the electric field integral equa-tion are available [5] as well, and will be discussed briefly.

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Coupling of finite and boundary elements for transient eddy current problems

Dalibor Lukáš
TU VSB Ostrava, Czech Republic

A symmetric coupling of methods of finite and boundary elements for numerical solution of transient eddy current problems will be presented. This is an essential step in modelling of electromagnetic forming of metalic sheets. The finite element method is employed in the conducting region. The boundary element method relies on the Stratton-Chu representation formula and it models the electromagnetic field in the air including its decay at infinity. We impose external currents by the Biot-Savart law. The eddy current simulations are postprocessed with finite element transient heat simulations and elastodynamics of the metalic sheet.

On the non-symmetric FEM BEM coupling for the Stokes problem

Günther Of
TU Graz, Austria

In recent years, there has been substantial progress on the stability analysis of discrete systems of the so-called non-symmetric coupling of FEM and BEM for Lipschitz domains. In this setting a finite element discretization is used for one subdomain while the weakly singular boundary integral equation is considered for a second subdomain. This approach leads to non-symmetric discrete linear systems. In this talk, we will discuss the non-symmetric coupling for the Stokes problem, which includes additional challenges, and we will present related numerical examples.

Time-domain boundary elements for elastodynamic trusses

Dominik Pölz[1], Michael Helmut Gfrerer[2], Martin Schanz[1]
[1]TU Graz, Austria, [2]Universität Kaiserslautern, Germany

We consider truss structures which consist of several elastic rods. In each of these members, the dynamic behaviour is governed by the 1D wave equation. The members are connected to each other by kinematic and kinetic coupling conditions. We present associated time-domain boundary integral equations based on an explicit and computable representation of the Dirichlet-to-Neumann map. Galerkin discretizations of these equations are discussed and we provide a criterion on the time step size which is sufficient for their stability. Adaptive mesh refinement is enabled by an a posteriori error indicator. The implementation is verified by numerical experiments. Finally, a typical truss system of structural engineering is simulated, illustrating the potential of the proposed method.

Solving integral equations for electromagnetic scattering using Bempp

M. Scroggs, T. Betcke, E. Burman, W. Śmigaj, E. van 't Wout
University College London, UK

The numerical simulation of electromagnetic wave scattering poses significant theoretical and computational challenges. Much effort in recent years has gone into the development of fast and robust boundary integral equation formulations to simulate a range of phenomena from the design and performance of antennas to radar scattering from large metallic objects.
While there have been a range of important theoretical advances in recent years for the development of robust preconditioned boundary integral formulations for Maxwell, the computational implementation remains a challenge. At University College London, as part of the Bempp project, we have developed a number of easy to use Python-based open-source tools to explore and solve Maxwell problems based on preconditioned electric field (EFIE), magnetic field (MFIE) and combined field (CFIE) integral equation formulations.
In this talk, we will give an overview of these formulations using an example exterior wave scattering problem. We will look at the importance of carefully selecting the finite dimensional spaces in order to produce a stable discretisation of the formulations, and compare the formulations using a number of interesting, non-trivial domains.

Coercive space–time finite and boundary element methods

Olaf Steinbach
TU Graz, Austria

For time-dependent partial differential equations such as the heat or wave equation we discuss variational formulations which turn out to satisfy a related stability condition, or an equivalent ellipticity estimate. We provide a stability and error analysis and we present some numerical results which confirm the theoretical findings.
The talk is based on joint work with M. Zank.

Adaptive and higher-order time domain boundary elements for the wave equation

H. Gimperlein[1], D. Stark[1], E. P. Stephan[2]

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We present h and p-versions of the time domain boundary element method for boundary and screen problems for the wave equation in $\mathbb{R}^3$. First, graded meshes are shown to recover optimal approximation rates for solution in the presence of edge and corner singularities on screens. Then an a posteriori error estimate is presented for general discretizations, and it gives rise to adaptive mesh refinement procedures. We also discuss preliminary results for p and hp-versions of the time domain boundary element method. Numerical experiments illustrate the theory.

Operator preconditioning for the electric field integral equation on the disk

Ralf Hiptmair, Carolina Urzúa-Torres
ETH Zürich, Switzerland

We consider the electric field integral equation (EFIE) arising from the scattering of time-harmonic electromagnetic waves by a perfectly conducting disk. When discretizing the EFIE by means of Galerkin boundary element methods (BEM), we obtain ill-conditioned systems on fine meshes. Consequently, preconditioning is needed and, due to the edge singularities featured by the solution of the EFIE on disks, its amenability to adaptive refinement is desirable.
The standard "Calderón preconditioning" technique is suboptimal when dealing with open surfaces like the disk [1]. In addition, it requires a div-conforming dual finite element space such that the curl/div duality pairing matrix is well conditioned. The exhisting technique resorts to BC functions [2] to fulfill this property on uniform meshes. However, the resulting dual pairing matrix becomes ill-conditioned as the ratio $h_{\max}/h_{\min}$ increases and demands additional diagonal scaling in order to handle non-uniform meshes.
In this presentation, we discuss a new strategy to build a preconditioner for the EFIE on the disk using operator preconditioning. Our approach uses recently found Calderón-type identities on the disk [3] and expects h-independent condition numbers. Furthermore, our discretization guarantees stability by means of the same dual finite element space one uses in the case of the Laplacian and allows for non-uniform meshes without additional computational effort.

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Space-time methods for the wave equation

Olaf Steinbach, Marco Zank
TU Graz, Austria

For the discretisation of time-dependent partial differential equations usually explicit or implicit time stepping schemes are used. An alternative approach is the usage of space-time methods, where the space-time domain is discretised and the resulting global linear system is solved at once. In this talk the model problem is the scalar wave equation. First, a brief overview of known results for the wave equation is presented. Second, a space-time formulation is motivated and discussed.
Finally, numerical examples for a one-dimensional spatial domain are presented.

Efficient analytic evaluation of boundary integral operators

Jan Zapletal[2], Günther Of[1], Michal Merta[2], [2]Lukas Maly
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Contrary to the volume-based finite element methods, a considerable amount of computational time in boundary element methods (BEM) is spent on the assembly of system matrices. Not only is this true for the fully populated classical BEM matrices, but also for the matrices sparsified by the fast multipole method or the adaptive cross approximation. Thus, an efficient implementation is required. In the talk we concentrate on two aspects of the assembly, namely on the simultaneous analytic evaluation of boundary integral operators for constant and linear shape functions and on the parallel and vectorized implementation suitable for modern multi- and many-core processors with wide SIMD registers.
Although the fully numerical regularized quadrature scheme is available for a broad range of kernels, the semi-analytical evaluation of Galerkin matrices has its advantages including its higher precision and a rather low number of quadrature nodes for the outer surface integral. On the other hand, in the original approach the operator had to be evaluated separately for a constant function and for every linear shape function supported on a triangular element which lead to a high number of expensive evaluations of transcendental functions including exp, log, or atan. Since these separate evaluations make use of similar terms, we propose a unified evaluation for all four shape functions. This approach naturally leads to a faster assembly of matrices with linear ansatz functions, but can also be exploited in BEM operator preconditioning when certain matrices have to be assembled with the same kernel but different test and ansatz function spaces.
In addition, we present a parallel implementation of the suggested routines. The local element contributions are distributed to individual threads by standard OpenMP pragmas. In addition, the single-instruction-multiple-data (SIMD) vectorization is achieved by refactorization of the original code and by the SIMD pragmas available in the OpenMP 4.5 standard. We present performance experiments carried out on the multi-core Haswell architecture and two generations of the many-core Intel Xeon Phi chips, namely the Knights Corner and Knights Landing. The provided results validate both the suggested simultaneous evaluation and its parallelized and vectorized implementation.

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