Technische Universität Graz

19. Workshop on

Fast Boundary Element Methods in Industrial Applications

Söllerhaus, 17.–20.10.2021

U. Langer, M. Schanz, O. Steinbach, W. L. Wendland (eds.)

Berichte aus dem Institut für Angewandte Mathematik

Book of Abstracts 2021/15

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Technische Universität Graz
Institut für Angewandte Mathematik
Steyrergasse 30
A 8010 Graz

WWW: `http://www.applied.math.tugraz.at`

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Program

Sunday, October 17, 2021	
15.00	Coffee
16.30–17.00	Opening
17.00–17.30	S. Chandler–Wilde (Reading) On the convergence of Galerkin BEM for classical 2nd kind boundary integral equations in Lipschitz domains
17.30–18.00	M. Zank (Wien) Coercive space-time single layer operator of the wave equation for flat objects
18.00–18.30	C. Urzua–Torres (Delft) Calderon preconditioning for the heat equation over triangular meshes
18.30	Dinner
Monday, October 18, 2021	
8.00–9.00	Breakfast
9.00–9.30	W. Wendland (Stuttgart) Propagation of acoustic waves in a thermo-electro-magneto-elastic solid
9.30–10.00	M. Kohr (Cluj) Layer potentials for the anisotropic Stokes system and applications
10.00–10.30	E. Spence (Bath) Coercive second-kind boundary integral equations for the Laplace Dirichlet problem on Lipschitz domains
10.30–11.00	Break
11.00–11.30	C. Schwab (Zürich) Deep neural network BEM
11.30–12.00	M. Multerer (Lugano) Data compression with samplets
12.00–12.30	R. von Rickenbach (Basel) Isogeometric shape optimisation for scaffold structures
12.30	Lunch
15.00	Coffee
15.30–16.00	H. Gimperlein (Edinburgh) Coupled finite and boundary elements for strongly nonlinear transmission problems
16.00–16.30	G. Unger (Graz) Coupled finite and boundary element methods for electromagnetic scattering-resonance problems
16.30–17.00	M. Feischl (Wien) Optimality of adaptive FEM-BEM coupling
17.00–17.30	Break
17.30–18.00	A. Buchau (Stuttgart) Series expansions of spherical harmonics and their application to electric and magnetic field problems
18.00–18.30	G. Di Credico (Parma) ACA based acceleration of the energetic Galerkin BEM for 2D acoustic and elastic wave propagation problems
18.30	Dinner

Tuesday, October 19, 2021	
8.00–9.00	Breakfast
9.00–9.30	V. Nistor (Metz) A Green function method for parabolic equations
9.30–10.00	G. Gantner (Amsterdam) Adaptive space-time BEM for the heat equation
10.00–10.30	R. Watschinger (Graz) A proof of an integration by parts formula for the bilinear form of the hypersingular operator of the heat equation
10.30–11.00	Break
11.00–11.30	M. Merta (Ostrava) Parallel implementation of the fast multipole method for the heat equation
11.30–12.00	P. Marchand (Bath) High-frequency estimates on boundary integral operators for the Helmholtz exterior Neumann problems
12.00–12.30	E. Schulz (Zürich) First-kind boundary integral equations for Hodge-Dirac operators and the trace de Rham complex
12.30	Lunch
13.30–18.00	Hiking Tour
18.30	Dinner

Wednesday, October 20, 2021	
8.00–9.00	Breakfast
9.00–9.30	R. Hiptmair (Zürich) Spurious quasi-resonances
9.30–10.00	H. Harbrecht (Basel) Isogeometric multilevel quadrature for forward and inverse random acoustic scattering
10.00–10.30	U. Langer (Linz) Adaptive space-time finite element methods for parabolic optimal control problems
10.30–11.00	O. Steinbach (Graz) The Hilbert transformation

Series expansions of spherical harmonics and their application to electric and magnetic field problems

A. Buchau
Universität Stuttgart

Spherical harmonics and series expansions based on spherical harmonics are well-known functions in physics and engineering disciplines. E.g. they are the solution of Schrödinger's equation of the hydrogen atom. Hence, especially in quantum mechanics, properties of these functions have been studied and transformations of spherical harmonics due to a change of the coordinate system have been developed. This has been lead among other things to the development of the Clebsch-Gordan coefficients and the Wigner 3j-symbol. Another classical application of spherical harmonics is the solution of boundary value problems described using the Poisson equation and a separation approach in spherical coordinates. Then, Green's function of 3d Poisson's equation is approximated by a truncated series expansion based on spherical harmonics. In the case of static electric field problems, the coefficients of this series expansion can be interpreted as multipoles. A combination of series expansions of spherical harmonics and the transformations that have been introduced in quantum mechanics results in the fast multipole method which is an established matrix compression technique for the boundary element method. Interestingly, spherical harmonics are a standard tool to model magnetic fields in the context of the shimming of magnets for magnetic nuclear resonance spectroscopy applications, too. There, spherical harmonics are used not in dependency of spherical coordinates but directly in Cartesian coordinates.

Here, the historical development of spherical harmonics is revisited. Then, I put the focus on some properties of the series expansions of spherical harmonics, especially I show numerically that the terms of the series expansion of spherical harmonics depend on spherical coordinates but the described function depends on Cartesian coordinates. For low order terms, I will show simple closed-form expressions in Cartesian coordinates which are e. g. used for many years in magnet shimming applications. Furthermore, a systematic error analysis of transformations of spherical harmonics is shown exemplarily for the reversed multipole algorithm. Finally, I will summarize the properties of spherical harmonics and will give an outlook to further applications of these powerful functions for applications in electrical engineering.

On the convergence of Galerkin BEM for classical 2nd kind boundary integral equations in Lipschitz domains

S. Chandler-Wilde[1], E. Spence[2]
[1]University of Reading, UK, [2]University of Bath, UK

The classical second kind integral equations for problems in potential theory can be written in operator form as $A\phi = g$ or as $A^*\psi = h$, where $A = \frac{1}{2}I \pm D$, I is the identity operator, D is the classical double-layer potential operator on the boundary Γ of the domain, and A^* is the $L^2(\Gamma)$ adjoint of A. Thanks to O. Steinbach and W. L. Wendland (J. Math. Anal. Appl., 2001) D has essential norm $< \frac{1}{2}$ as an operator on $H^{1/2}(\Gamma)$, which implies that Galerkin methods in $H^{1/2}(\Gamma)$ and in $H^{-1/2}(\Gamma)$ converge for $A\phi = g$ and $A^*\psi = h$, respectively. Long-standing open questions (e.g., W. L. Wendland, 'On the Double Layer Potential', in *Analysis, Partial Differential Equations and Applications*, Springer, 2009) are whether, at least in 2d, D has essential norm $< \frac{1}{2}$ as an operator on $L^2(\Gamma)$, or whether, at any rate, the weaker property holds that A and A^* are the sum of coercive and compact operators as operators on $L^2(\Gamma)$. Summarising recent work by the authors (arXiv:2105.11383, 2021) we present examples that show that this is not the case. Precisely, we present examples, in 2d and 3d, of Lipschitz domains with Lipschitz constant one for which, as an operator on $L^2(\Gamma)$, $\|D\|_{\mathrm{ess}} \geq 1/2$, and examples with Lipschitz constant two for which D is not coercive plus compact. We also exhibit counterexamples that are starshaped polyhedra. We show that this implies, for all these counterexamples, that there exist $L^2(\Gamma)$ Galerkin BEMs that are not convergent.

ACA based acceleration of the Energetic Galerkin BEM for 2D acoustic and elastic wave propagation problems

A. Aimi, L. Desiderio, G. Di Credico
University of Parma, Italy

We consider acustic and elastic wave propagation problems in 2D unbounded domains, re-formulated in terms of space-time Boundary Integral Equations (BIEs). For their numerical resolution, we employ a weak formulation depending on the energy of the system and we solve the related discretized problem by a Galerkin Boundary Element Method (BEM). This approach founds its best application in the study of external diffusion phenomena and is useful to overcome the instabilities rising from the discretization of the standard weak formulation applied to this kind of integral problems [1,2]. However, it is necessary to take into account that, when standard Lagrangian basis functions are considered, the BEM matrices have time blocks that become in general fully populated, and the overall memory cost of the energetic BEM is $\mathcal{O}(M^2N)$, M and N being the number of grid points chosen on the domain boundary and the total number of time steps performed, respectively. This drawback prevents the application of such method to large scale realistic problems.
For this reason, our purpose is to provide a fast technique, based on the Adaptive Cross Approximation (ACA) [3], with which we get a low rank approximation of sufficiently large time blocks of the energetic boundary element matrix, reducing drastically the number of the original entries to be evaluated. This leads to a drop in the assembly time and in the memory storage requirements, which are generally relevant. Moreover, the consequent acceleration of the matrix/vector multiplication together with a marching on time procedure, leads remarkable reduction of the computational solution time. The effectiveness of the proposed method is theoretically proved and several numerical results are presented and discussed.

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Optimality of adaptive FEM-BEM coupling

M. Feischl
TU Wien, Austria

We propose a generalization of quasi-orthogonality which follows directly from the inf-sup stability of the underlying problem. This completely removes a central technical difficulty in modern proofs of optimal convergence of adaptive mesh refinement algorithms and leads to a simple optimality proof of a finite-element/boundary-element discretization of an unbounded transmission problem. The main technical tools are new stability bounds for the LU-factorization of matrices together with a recently established connection between quasi-orthogonality and matrix factorization.

Adaptive space-time BEM for the heat equation

G. Gantner, R. van Venetië
University of Amsterdam, The Netherlands

In this talk, which is based on our recent work [1], we consider the space-time boundary element method [2,3] for the heat equation with prescribed initial and Dirichlet data. One often mentioned advantage of simultaneous space-time methods is their potential for fully adaptive refinement to resolve singularities local in both space and time. To this end, we propose a residual-type *a posteriori* error estimator similar to [4,5] that is a lower bound and, up to weighted L_2-norms of the residual, also an upper bound for the unknown BEM error. The possibly locally refined meshes are assumed to be prismatic, i.e., their elements are tensor-products $J \times K$ of elements in time J and space K. While the results do not depend on the local aspect ratio between time and space, assuming the scaling $|J| \simeq \mathrm{diam}(K)^2$ for all elements and using Galerkin BEM, the estimator is shown to be efficient and reliable without the a dditional L_2-terms. In the considered numerical experiments on two-dimensional domains in space, the estimator seems to be equivalent to the error, independently of these assumptions. In particular for adaptive anisotropic refinement, both converge with the best possible convergence rate.

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Coupled finite and boundary elements for strongly nonlinear transmission problems

H. Gimperlein[1], M. Maischak[2], E. P. Stephan[3]

[1]Heriot-Watt University, Edinburgh, UK
[2]Brunel University, Uxbridge, UK
[3]Leibniz Universität Hannover, Germany

This talk discusses the well-posedness and error analysis of coupled finite and boundary elements for transmission or contact problems. Nonlinear operators like the scalar p-Laplacian or nonlinear Hencky materials with an unbounded stress-strain relation arise in the modelling of ice sheets, non-Newtonian fluids or porous media. We consider interface problems which couple such operators to the linear Laplace equation, resp. linear elasticity, in the exterior. The exterior problem is reduced to the boundary using a symmetric coupling formulation for the Poincare-Steklov operator. We present a functional analytic framework for the numerical analysis of the resulting boundary/domain variational problem. A priori and a posteriori error estimates are obtained for Galerkin approximations. Numerical experiments underline the theoretical results.

Isogeometric multilevel quadrature for forward and inverse random acoustic scattering

H. Harbrecht

Universität Basel, Switzerland

We study the numerical solution of forward and inverse acoustic scattering problems by randomly shaped obstacles in three-dimensional space using a fast isogeometric boundary element method. Within the isogeometric framework, realizations of the random scatterer can efficiently be computed by simply up- dating the NURBS mappings which represent the scatterer. This way, we end up with a random deformation field. In particular, we show that the knowledge of the deformation field's expectation and covariance at the surface of the scatterer are already sufficient to model the surface Karhunen-Loève expansion. Leveraging on the isogeometric framework, we utilize multilevel quadrature methods for the efficient approximation of quantities of interest, such as the scattered wave's expectation and variance. Computing the wave's Cauchy data at an artificial, fixed interface enclosing the random obstacle, we can also directly infer quantities of interest in free space. Adopting the Bayesian paradigm, we finally compute the expected shape and the variance of the scatterer from noisy measurements of the scattered wave at the artificial interface. Numerical results for the forward and inverse problem are given to demonstrate the feasibility of the proposed approach.

Spurious quasi-resonances

R. Hiptmair[1], A. Moiola[2], E. A. Spence[3]

[1]ETH Zürich, Switzerland, [2]University of Pavia, Italy, [3]University of Bath, UK

We consider the Helmholtz transmission problem with piecewise-constant material coefficients, and the standard associated direct boundary integral equations. For certain coefficients and geometries, the norms of the inverses of the boundary integral operators grow rapidly through an increasing sequence of frequencies, even though this is not the case for the solution operator of the transmission problem; we call this phenomenon that of spurious quasi-resonances. We give a rigorous explanation of why and when spurious quasi-resonances occur, and propose modified boundary integral equations that are not affected by them.

Layer potentials for the anisotropic Stokes system and applications

M. Kohr

The aim of this talk is to describe a layer potential theory in L_2-based weighted Sobolev spaces on Lipschitz bounded and exterior domains of $\mathbb{R}^n$, $n \geq 3$, for the anisotropic Stokes system with L_∞ elliptic viscosity tensor coefficient satisfying an ellipticity condition for symmetric matrices with zero matrix trace. We explore equivalent mixed variational formulations and prove the well-posedness of some transmission problems for the anisotropic Stokes system in Lipschitz domains of $\mathbb{R}^n$, with the given data in L_2-based weighted Sobolev spaces. These results are used to define the volume (Newtonian) and layer potentials and to obtain their properties. Then we analyze the well-posedness of the exterior Dirichlet and Neumann problems for the anisotropic Stokes system by representing their solutions in terms of the obtained volume and layer potentials. Boundary value problems for the anisotropic Navier-Stokes system will be also discussed.
Joint work with Sergey E. Mikhailov (Brunel University London) and Wolfgang L. Wendland (Stuttgart University).

Adaptive space-time finite element methods for parabolic optimal control problems

U. Langer[1], A. Schafelner[1], O. Steinbach[2], F. Tröltzsch[3], H. Yang[1]

[1]Johannes Kepler Universität Linz, Austria, [2]TU Graz, Austria, [3]TU Berlin, Germany

We consider tracking-type optimal control problems constrained by linear parabolic partial differential equations with distributed source control. Here, space-time finite element methods on unstructured simplicial meshes are especially suited, since the reduced optimality system couples two parabolic equations for the state and adjoint state that are forward and backward in time, respectively. The analysis of the reduced optimality system and discretization error estimates are based on Banach-Nečas-Babuška-Aziz's theorems. In contrast to time-stepping methods, one has to solve one large-scale linear algebraic system of finite element equations the solution of which provides continuous finite element approximations to the state and adjoint state in the whole space-time cylinder at once. Full space-time adaptivity, parallelization, and matrix-free implementations are very important techniques to overcome the increased memory requirement of space-time finite element methods. Fast parallel solvers are another important ingredient of efficient space-time methods. We first consider the standard L_2 regularization, and then compare it with sparse optimal control techniques and a new energy regularization. The numerical results confirm the convergence rate estimates in the case of uniform refinement, the efficiency of the adaptivity procedure proposed, and they clearly show the effects of different regularization techniques.
The space-time approach proposed in the talk can be extended to other optimal control problems like partial control, boundary control, initial date control, partial observation, terminal observation, but also to non-linear parabolic state equations and box constraints imposed on the control.

High-frequency estimates on boundary integral operators for the Helmholtz exterior Neumann problem

J. Galkowski[1], P. Marchand[2], E. A. Spence[2]
[1]University College London, UK, [2]University of Bath, UK

We study a commonly-used second-kind boundary-integral equation for solving the Helmholtz exterior Neumann problem at high frequency, namely the Regularized Combined Field Integral Equation (RCFIE) introduced in [1]. Writing Γ for the boundary of the obstacle, this integral operator map $L^2(\Gamma)$ to itself, contrary to its non-regularized version.
We prove new frequency-explicit bounds on the norms of both the RCFIE and its inverse. The bounds on the norm are valid for piecewise-smooth Γ and are sharp, and the bounds on the norm of the inverse are valid for smooth Γ and are observed to be sharp at least when Γ is curved. Together, these results give bounds on the condition number of the operator on $L^2(\Gamma)$; this is the first time $L^2(\Gamma)$ condition-number bounds have been proved for this operator for obstacles other than balls [2].

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Parallel implementation of the fast multipole method for the heat equation

M. Merta[1], G. Of[2], R. Watschinger[2], J. Zapletal[1]

[1] TU VSB Ostrava, Czech Republic, [2]TU Graz, Austria

In this talk we present a novel approach to the parallelization of the fast multipole method (FMM) for a space-time boundary element method for the heat equation in three spatial dimensions. Our approach employs a scheduler for the MPI-based distributed memory parallelization which takes into account the special temporal structure of the involved operators. The original space-time FMM is associated with a 1D temporal tree that is distributed among computing processes and allows to group actual FMM operations in time. Moreover, a task-based shared memory parallelization and SIMD vectorization is applied to fully leverage the computational power of modern supercomputers. The presented numerical experiments demonstrate a high parallel efficiency of the code up to hundreds of compute nodes.

Data compression with samplets

M. Multerer
Universita della Svizzera italiana, Lugano, Switzerland

In this talk, we introduce the concept of samplets by transferring the construction of Tausch-White wavelets to the realm of data. This way we obtain a multilevel representation of discrete data which directly enables data compression, detection of singularities and adaptivity. Applying samplets to represent kernel matrices, as they arise in kernel based learning or Gaussian process regression, we end up with quasi-sparse matrices. By thresholding small entries, these matrices are compressible to $O(N \log N)$ relevant entries, where N is the number of data points. This feature allows for the use of fill-in reducing reorderings to obtain sparse factorizations of the compressed matrices. Besides the introduction of samplets and the discussion of their properties, we present numerical studies to benchmark the approach.

A Green function method for parabolic equations

V. Nistor
Institut Élie Cartan de Lorraine, Metz, France

We propose a method to approximate the fundamental solution of a uniformly parabolic PDE on euclidean space. We show how this approximation can be used for the approximation of the Initial Value Problem for such a PDE. Joint work with Mazzucato.

Isogeometric shape optimisation for scaffold structures

R. von Rickenbach
Universität Basel, Switzerland

The development of materials with specific structural properties is of huge practical interest, for example, for medical applications or for the development of lightweight structures in aeronautics. In this article, we combine shape optimisation and homogenisation for the optimal design of the microstructure in scaffolds. Given the current microstructure, we apply the isogeometric boundary element method to compute the effective tensor and to update the microstructure by using the shape gradient in order to match the desired effective tensor. The deformation basis is constructed via the Karhunen-Loève expansion of a covariance kernel, for instance, a Matérn kernel. Extensive numerical studies are presented to demonstrate the applicability and feasibility of the approach.

First-kind boundary integral equations for Hodge–Dirac operators and the trace de Rham complex

E. Schulz

ETH Zürich, Switzerland

We develop novel first-kind boundary integral equations for Hodge–Dirac operators in Lipschitz domains comprising square-integrable potentials and involving only weakly singular kernels. Generalized Garding inequalities are derived and we establish that the obtained boundary integral operators are Fredholm of index zero. Their finite dimensional kernels are characterized and we show that their dimension is equal to the number of topological invariants of the domain's boundary, in other words to the sum of its Betti numbers. This is explained by the fundamental discovery that the associated bilinear forms agree with those induced by surface Dirac operators for $H^{-1/2}$ surface (de Rham) Hilbert complexes whose underlying inner-products are the non-local inner products defined through the classical single-layer boundary integral operators for the Laplacian. Decay conditions for well-posedness in natural energy spaces of the Dirac system in unbounded exterior domains are also presented.

Deep neural network BEM

C. Schwab
ETH Zürich, Switzerland

We introduce a Neural Network (NN for short) approximation architecture for the numerical solution of BIEs. We adopt a Galerkin formulation with 1st kind BIEs on polygonal domains with a finite number of straight sides. Trial spaces used in the Galerkin discretization of the BIEs are built by using NNs that, in turn, employ the so-called Rectified Linear Units (ReLU).

The ReLU-NNs used to approximate the solutions to the BIEs depend nonlinearly on the parameters characterizing the NNs themselves. The computation of a numerical solution to a BIE by means of ReLU-NNs boils down to a fine tuning of these parameters, in *network training*.

We argue that by using ReLU-NNs with various combinations of width and depth as Galerkin trial spaces, one essentially recovers well-known approximation results for the standard Galerkin Boundary Element Method (BEM). We propose to employ well-known *a posteriori* error estimators to build local and efficiently computable loss functions to train the ReLU-NNs for the numerical approximation of BIEs.

Exploratory numerical experiments validate our theoretical findings and indicate the viability of the proposed ReLU-NN Galerkin BEM approach.

Joint work with F. Henriquez (EPFL) and R. Aylwin (UAI, Chile).

Coercive second-kind boundary integral equations for the Laplace Dirichlet problem on Lipschitz domains

S. Chandler-Wilde[1], E. Spence[2]

[1]2University of Reading, UK, [2]University of Bath, UK

Recent work by the authors (arXiv:2105.11383, 2021), recapped in the talk by Simon Chandler-Wilde, has show that there exists star-shaped Lipschitz polyhedra for which the boundary-integral operators $\pm\frac{1}{2}I+D$ and $\pm\frac{1}{2}I+D'$ (where D is the Laplace double-layer operator and D' its adjoint) cannot be written as the sum of a coercive operator and a compact operator in $L^2(\Gamma)$ (where Γ is the boundary). This implies that, for these domains and operators, there exist non-convergent Galerkin BEMs in $L^2(\Gamma)$.

A natural question is then: *do there exist second-kind boundary-integral formulations in* $L^2(\Gamma)$ *of Laplace's equation where, with* Γ *only assumed to be Lipschitz, the operators are continuous, invertible, and can be written as the sum of a coercive operator and a compact operator?*

This talk answers this question in the affirmative for the Laplace interior and exterior Dirichlet problems. We present new BIE formulations that are continuous and *coercive* (i.e., not just the sum of a coercive and a compact operator) in $L^2(\Gamma)$, with Γ only assumed to be Lipschitz; thus the Galerkin method in $L^2(\Gamma)$ converges for every asymptotically-dense sequence of subspaces. Furthermore, the strong property of coercivity allows us to prove that if the Galerkin matrices are preconditioned by a specified diagonal matrix, then the number of GMRES iterations required to solve the associated linear systems to a prescribed accuracy does *not* increase as the discretisation is refined and N increases; i.e., the new formulations exhibit the same conditioning properties as the standard second-kind BIEs on $L^2(\Gamma)$ when Γ is C^1.

The Hilbert transformation

O. Steinbach
TU Graz, Austria

In [6,7] we have introduced a modified Hilbert transformation $\mathcal{H}_T$ such that

$$\langle \partial_t v, \mathcal{H}_T v \rangle_{(0,T)} = \|v\|^2_{H^{1/2}_{0,}(0,T)}$$

defines a norm for $v \in H^{1/2}(0,T)$ with $v(0) = 0$. Using this modified Hilbert transformation we were able to derive coercive space-time variational formulations not only for the heat equation [6], but also for the wave equation [1,4,5,8]. While the application of the modified Hilbert transformation may be replaced by the classical Hilbert transformation $\mathcal{H}$ [2], we will show that the modified Hilbert transformation can be written as the classical one including a suitable extension from $(0,T)$ onto $\mathbb{R}$, with a compact perturbation [3].

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Coupled finite and boundary element methods for electromagnetic scattering-resonance problems

G. Unger

TU Graz, Austria

In this talk we study the convergence of coupled finite and boundary element methods for electromagnetic scattering-resonance problems for dielectric and plasmonic scatterers and show numerical results of the implementation using the open-source MATLAB toolbox `Gypsilab` [1].
We consider a so-called symmetric formulation of the resonance problem which is based on a coupling of a weak formulation of Maxwell's equations inside the scatterer with the Calderón projector for Maxwell's equations outside the scatterer. For the related source problem this kind of formulation and its discretization was already analyzed in [2]. It was shown that for positive frequencies and positive material parameters this kind of formulation for the source problem is weakly T-coercive and that a discretization with Nedelec elements inside the scatterer together with Raviart-Thomas boundary elements on the surface of the scatterer yields quasi-optimal convergence. We extend these results with respect to the weak T-coercivity to complex-valued frequencies and to complex-valued material parameters by introducing a frequency and material dependent operator T. The convergence of the Galerkin approximation of the resonance problem is shown by combing recent results on the regular approximation of weakly T-coercive operators [3,6] with classical results on the approximation of eigenvalue problems for holomorphic Fredholm operator-valued functions [4,5]. Numerical experiments confirms the theoretical results.

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Calderón preconditioning for the heat equation over triangular meshes

C. Urzúa-Torres, D. Hoonhout
Delft University of Technology, Netherlands

In this talk, we report on two computational aspects of space-time boundary element methods for the heat equation over triangular meshes. First, under certain assumptions over the mesh, we compute the boundary element matrices using a semi-analytic integration scheme where the temporal integrals are treated analytically, similarly to what is done when using tensor product meshes. Then, we present Calderón preconditioning using the discretization proposed by Stevenson and van Venetïe [1], instead of the classical primal-dual mesh approach. This offers the advantage of having a duality coupling matrix which is diagonal and thus better suited for space-time parallelization.

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A proof of an integration by parts formula for the bilinear form of the hypersingular operator of the heat equation

R. Watschinger, G. Of
TU Graz, Austria

When considering the hypersingular boundary integral operator D in applications, one has to find a way to evaluate the occurring integrals despite the strong singularity of the involved integral kernel. In case of Galerkin methods there exist integration by parts formulas for a wide class of PDEs which allow to represent the bilinear form induced by D by some weakly singular bilinear forms. Also for the transient heat equation in three spatial and one temporal dimensions such a formula is given in the literature, but a proof seemed to be missing. Moreover, the available integration by parts formula contains an integral term involving the temporal derivative of the fundamental solution of the heat equation, which is not integrable in the considered integration domain. To fill these gaps, we provided a rigorous proof of the integration by parts formula and an alternative representation of the mentioned integral term in [1]. In this talk we present the main ideas of the proof and shortly discuss the new representation.

References

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Propagation of acoustic waves in a thermo-electro-magneto-elastic solid

G. C. Hsiao[1], W. L. Wendland[2]

[1]University of Delaware, USA, [2]Universität Stuttgart, Germany

We are concerned with a time–dependent transmission problem for a thermo–electric–magneto–elastic solid immersed in an inviscid and compressible fluid. This problem can be treated by a boundary–field equation method, provided an appropriate scaling factor is employed. Based on estimates of variational solutions in the Laplace–transformed domain we obtain with explicit inversions of the variational solution a convolutional form in the Laplace–transformed domain and also the time–dependent representation.

Coercive space-time single layer operator of the wave equation for flat objects

M. Costabel[1], M. Zank[2]

[1]Université de Rennes 1, France, [2]Universität Wien, Austria

The solution of the wave equation can be described by a single layer representation. First, an overview of known results for this single layer representation and the boundary integral equations for the wave equation is given. Second, in the case of a flat screen, a new approach is introduced. For this purpose, we apply the Fourier transformation to the single layer operator with respect to space and time. This leads to a Fourier representation of the single layer operator. Further, this Fourier representation motivates to apply the (classical) Hilbert transformation to the single layer operator. Summing up both components and introducing new space-time Sobolev spaces, this approach leads to a coercive and continuous single layer operator. Finally, properties of these new space-time Sobolev spaces and the single layer operator in these spaces are given.

Participants

1. Dr. Martin Averseng
   Seminar für Angewandte Mathematik, ETH Zürich, Switzerland
   martin.averseng@gmail.com

2. Dr. Pieter Barendrecht
   Departement Mathematik & Informatik, Universität Basel, Switzerland
   pieterjaap.barendrecht@unibas.ch

3. Dr.–Ing. Andre Buchau
   Institut für Intelligente Sensorik und Theoretische Elektrotechnik, Universität Stuttgart, Germany
   andre.buchau@iis.uni-stuttgart.de

4. Prof. Dr. Simon Chandler-Wilde
   Department of Mathematics and Statistics, University of Reading, UK
   S.N.Chandler-Wilde@reading.ac.uk

5. Giulia Di Credico, M.Sc.
   University of Parma, Italy
   giulia.dicredico@unipr.it

6. Prof. Dr. Michael Feischl
   Institute for Analysis and Scientific Computing, TU Wien, Austria
   michael.feischl@tuwien.ac.at

7. Dr. Gregor Gantner
   Korteweg-de Vries Instituut, University of Amsterdam, The Netherlands
   g.gantner@uva.nl

8. Prof. Dr. Heiko Gimperlein
   Department of Mathematics, Heriot–Watt University Edinburgh, UK
   H.Gimperlein@hw.ac.uk

9. Prof. Dr. Helmut Harbrecht
   Departement Mathematik & Informatik, Universität Basel, Switzerland
   helmut.harbrecht@unibas.ch

10. Prof. Dr. Ralf Hiptmair
    Seminar für Angewandte Mathematik, ETH Zürich, Switzerland
    ralfh@ethz.ch

11. Dr. Matthias Kirchhart
    Center for Computational Engineering Science, RWTH Aachen, Germany
    kirchhart@mathcces.rwth-aachen.de

12. Prof. Dr. Mirela Kohr
    Faculty of Mathematics and Computer Science, Babes-Bolyai University, Cluj, Romania
    mkohr@math.ubbcluj.ro

13. Ignacio Labarca, M.Sc.
    Seminar für Angewandte Mathematik, ETH Zürich, Switzerland
    ignacio.labarca@sam.math.ethz.ch

14. Prof. Dr. Ulrich Langer
    Institut für Numerische Matheatik, Johannes Kepler Universität Linz, Austria
    ulanger@numa.uni-linz.ac.at

15. Dipl.–Ing. Richard Löscher
Fachbereich Mathematik, TU Darmstadt, Germany
loescher@mathematik.tu-darmstadt.de

16. Prof. Dr. Dalibor Lukas
Department of Applied Mathematics, VŠB TU Ostrava, Czech Republic
dalibor.lukas@vsb.cz

17. Dr. Pierre Marchand
Department of Mathematical Sciences, University of Bath, UK
pfcm20@bath.ac.uk

18. Dr. Michal Merta
IT4Innovations, VŠB TU Ostrava, Czech Republic
michal.merta@vsb.cz

19. Prof. Dr. Michael Multerer
Institute of Computational Science, Universita della Svizzera italiana, Lugano, Switzerland
michael.multerer@usi.ch

20. Prof. Dr. Victor Nistor
Institut Élie Cartan de Lorraine, Université de Lorraine, Metz, France
victor.nistor@univ-lorraine.fr

21. Maximilian Nolte, M.Sc.
TEMF, TU Darmstadt, Germany
maximilian.nolte@stud.tu-darmstadt.de

22. Prof. Dr. Günther Of
Institut für Angewandte Mathematik, TU Graz, Austria
of@tugraz.at

23. Dr. Ceyhun Özdemir
Institut für Mechanik, TU Graz, Austria
oezdemir@tugraz.at

24. Piyush Panchal, M.Sc.
Seminar für Angewandte Mathematik, ETH Zürich, Switzerland
piyush.panchal@sam.math.ethz.ch

25. Remo von Rickenbach, M.Sc.
Departement Mathematik & Informatik, Universität Basel, Switzerland
remo.vonrickenbach@unibas.ch

26. Andrea Scaglioni, M.Sc.
Institute for Analysis and Scientific Computing, TU Wien, Austria
andrea.scaglioni@tuwien.ac.at

27. Erick Schulz, M.Sc.
Seminar für Angewandte Mathematik, ETH Zürich, Switzerland
erick.schulz@sam.math.ethz.ch

28. Prof. Dr. Christoph Schwab
Seminar für Angewandte Mathematik, ETH Zürich, Switzerland
christoph.schwab@sam.math.ethz.ch

29. Prof. Dr. Euan Spence
Department of Mathematical Sciences, University of Bath, UK
e.a.spence@bath.ac.uk

30. Prof. Dr. Olaf Steinbach
Institut für Angewandte Mathematik, TU Graz, Austria
o.steinbach@tugraz.at

31. David Ulcak, Ing.
Department of Applied Mathematics, VŠB TU Ostrava, Czech Republic
david.ulcak@vsb.cz

32. Dr. Gerhard Unger
Institut für Angewandte Mathematik, TU Graz, Austria
gerhard.unger@tugraz.at

33. Dr. Carolina Urzua Torres
Delft Institute of Applied Mathematics, TU Delft, Netherlands
c.a.urzuatorres@tudelft.nl

34. Dipl.–Ing. Raphael Watschinger
Institut für Angewandte Mathematik, TU Graz, Austria
watschinger@math.tugraz.at

35. Prof. Dr.–Ing. Wolfgang L. Wendland
Institut für Angewandte Analysis und Numerische Simulation,
Universität Stuttgart, Germany
wolfgang.wendland@mathematik.uni-stuttgart.de

36. Dr. Marco Zank
Fakultät für Mathematik, Universität Wien, Austria
marco.zank@univie.ac.at