ON SCHRÖDINGER OPERATORS WITH OBLIQUE TRANSMISSION CONDITIONS ON NON-SMOOTH CURVES

BADREDDINE BENHELLAL
CARL VON OSSIETZKY UNIVERSITÄT OLDENBURG

ABSTRACT. In this talk, I will present recent results on Schrödinger operators with transmission conditions along a Lipschitz (non-smooth) curves, extending previous work by Behrndt, Holzmann, and Stenzel on smooth curves. Given bounded domain $\Omega_+ \subset \mathbb{R}^2$ with a Lipschitz boundary $\Sigma \subset \mathbb{R}^2$ and a unit normal $N = (n_1, n_2)$, and define its complementary exterior domain Ω_-. For a parameter $\alpha \in \mathbb{R}$, the Schrödinger operator we consider acts as

$$(u_+, u_-) \mapsto (-\Delta u_+, -\Delta u_-) \quad \text{for } (u_+, u_-) \in L^2(\Omega_+) \oplus L^2(\Omega_-) \simeq L^2(\mathbb{R}^2),$$

with the so-called *oblique transmission condition*:

$$(n_1 + in_2)(\gamma^+ f_+ - \gamma^- f_-) + \alpha(\gamma^+ \partial_{\bar{z}} f_+ + \gamma^- \partial_{\bar{z}} f_-) = 0 \text{ on } \Sigma.$$

Earlier works focused on smooth curves, showing that these operators are self-adjoint, with well-understood spectral properties. In particular, the discrete spectrum is empty for $\alpha \geq 0$ and infinite and unbounded from below for $\alpha < 0$, without accumulation at 0, and for any fixed $n \in \mathbb{N}$ the n-th discrete eigenvalue λ_n (if counted with multiplicities in the non-increasing order) satisfies

$$\lambda_n = -\frac{4}{\alpha^2} + O(1) \text{ for } \alpha \to 0^-. \tag{0.1}$$

Our work generalizes these results to the case where the curve is only Lipschitz continuous. I will show that, while the operator remains self-adjoint and retains some of its spectral structure, the non-smoothness of the boundary introduces significant changes and the asymptotic (0.1) turns out to be false in general. Namely, using a relation with δ-potentials, we will show the two-sided estimate: for any $n \in \mathbb{N}$ there are $0 < A < B$ such that

$$-\frac{B}{\alpha^2} \leq \lambda_n \leq -\frac{A}{\alpha^2} \text{ for } \alpha \to 0^-.$$

Then, we will show that for any $B \in (1, 4)$ there exists a non-smooth curve Σ such that for the associated operator there holds

$$\lambda_1 = -\frac{B}{\alpha^2} + o\Big(\frac{1}{\alpha^2}\Big) \text{ for } \alpha \to 0^-,$$

which is clearly different from (0.1).

Based on joint work with Miguel Camarasa (BCAM, Bilbao) and Konstantin Konstantin (University of Oldenburg).