Exceptional Laguerre Differential Operators: their Formulation and Spectral Analysis

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Exceptional orthogonal polynomials (XOPs) emerged in 2009 from the study of exactly and quasi-exactly solvable potentials in quantum mechanics. Following the Bochner Classification Theorem in 1929, it was widely believed that the only complete sets of eigenfunctions for rational Sturm-Liouville equations were the classical orthogonal polynomial (COP) families of Hermite, Laguerre, and Jacobi. However, the discovery of XOPs broadened the set of admissible solutions. Surprisingly, despite "missing" certain polynomial degrees, XOPs form a complete set in their associated Hilbert space.

Each XOP family is linked to a corresponding COP family by a sequence of Darboux transformations. While these transformations are often described as "state-deleting" since a finite number of degrees are removed from the set of eigenfunctions, this terminology can be misleading when studying the spectrum. The set of eigenvalues for an XOP system may remain identical to that of its associated COP or even include new points!

In this talk, we will discuss the formulation of the Laguerre XOPs through Maya diagrams, a combinatoric tool used to illustrate the Darboux transformation process. Additionally, we will examine the consequences of these transformations on self-adjoint extensions of the differential expressions and the resulting spectrum.