Spectral properties of Robin-Laplacians on sharp infinite cones

Abstract

Let $\omega \subset \mathbb{R}^n$ be a bounded Lipschitz domain. For $\varepsilon > 0$ consider the infinite cone

$$\Omega_\varepsilon := \left\{ (x_1, x') \in (0, \infty) \times \mathbb{R}^n : x' \in \varepsilon x_1 \omega \right\} \subset \mathbb{R}^{n+1}$$

and the Laplace-Operator $Q_\varepsilon^\alpha u = -\Delta u$ with Robin boundary conditions $\partial_\nu u = \alpha u$ on $\partial\Omega_\varepsilon$, where ∂_ν is the outward normal derivative and $\alpha > 0$ is a coupling constant. We discuss several spectral properties of Q_ε^α. This includes the essential spectrum, the number of discrete eigenvalues and the bottom of the spectrum. After that we look at the asymptotic behavior of the eigenvalues $E_j(Q_\varepsilon^\alpha)$ when the cone gets sharper, i.e. $\varepsilon \to 0$.