Higher Optimal Eigenvalues on Metric Graphs

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In the first part of this talk, "Optimal eigenvalues on metric graphs", we studied the optimization of the first non-trivial eigenvalue of measure-dependent Laplacians on compact metric graphs. We now turn to higher eigenvalues and investigate how their optimal values and the corresponding minimizing measures depend on the geometry of the graph.

We show that every minimizing measure is supported on finitely many points, with an explicit upper bound on the size of its support. Consequently, the optimization problem reduces to a class of weighted Dirichlet-to-Neumann operators. We also establish universal eigenvalue bounds and asymptotic behavior analogous to Weyl's law.

For metric trees, we obtain a sharper result by establishing a connection to optimal spectral partitions. Using the nodal domain structure of eigenfunctions, we show that the kth optimal eigenvalue coincides with the minimal energy of a partition into $(k+1)$ connected subgraphs. This provides a geometric characterization of higher optimal eigenvalues on trees.