Oscillatory integrals and Evolution of Superoscillations

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One of the most famous examples of an oscillatory integral is the *Fresnel integral*

$$\int_{-\infty}^{\infty} e^{iy^2} dy. \tag{1}$$

An integral which obviously does not converge in the usual L^1-sense, but due to its fast oscillatory behaviour for large values of y, there still exist useful interpretations. In this talk I want to present a method of iterative integration by parts, to give meaning to integrals similar to (1), under rather minimal assumptions on the integrand.

A natural application of this theory is the *Time evolution of superoscillations*, where it is necessary to calculate the solution of the time dependent Schrödinger equation

$$\Psi(t,x) = \int_{\mathbb{R}} G(t,x,y)F(y)dy,$$

with plane wave initial conditions $F(y) = e^{iky}$. Since this functions do not admit any decay at infinity, Fresnel integral methods are needed to investigate those kind or problems.